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Journal

of

THE ROYAL AERONAUTICAL SOCIETY

SEPTEMBER 1953

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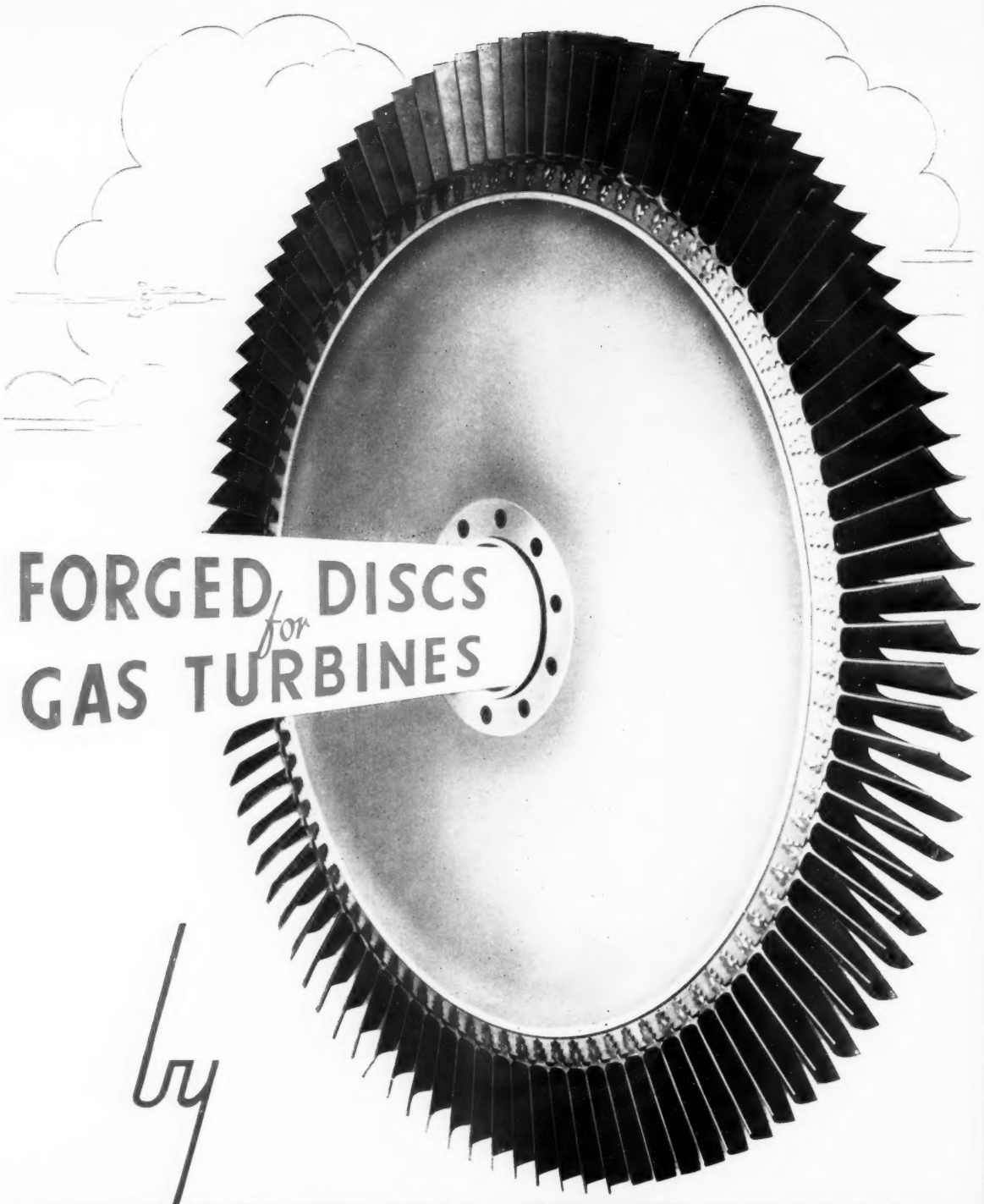

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All communications for publication in, and advertisements for, the Journal should be addressed to the Editor, Joan Bradbrooke, A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary, A. M. Ballantyne, T.D., B.Sc., Ph.D., A.F.R.Ae.S.

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless stated otherwise.

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(Founded 1866)

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NOTICES

Fourth Anglo-American Aeronautical Conference

There are so many delegates to the Fourth Anglo-American Aeronautical Conference, 15th-17th September, that larger accommodation has had to be found for the Lecture Sessions. University College will remain the headquarters of the Conference and lunch and other refreshments will be obtainable there each day, but the opening and closing ceremonies and all Lecture Sessions will be held at Friends House, Euston Road, a few minutes walk from University College. The programme published in the August Journal does not now apply and an amended time-table has been sent to all Delegates.

Members' New Appointments

R. A. Berridge (*Associate Fellow*), formerly of Aircraft Materials Ltd., has recently joined Hale, Hamilton & Co. Ltd., Uxbridge, as Technical Representative.

J. Brancker (*Associate*), formerly General Manager International Affairs, British Overseas Airways Corporation, has been appointed Traffic Director of the International Air Transport Association at Montreal.

T. Martin Chalmers (*Associate Fellow*) is relinguishing his post as a Lecturer in the University of Glasgow to take up an appointment on the 1st October as Assistant to the Technical Director of John Dalglish & Sons Ltd., Glasgow.

R. J. H. Isaacs (*Associate Fellow*) has been appointed Chief Stressman at the London Office of Blackburn & General Aircraft Ltd.

G. Moss (*Associate Fellow*) has been appointed Chief Draughtsman at the London Office of Blackburn & General Aircraft Ltd.

E. F. Priest (*Associate Fellow*) has been appointed Manager of the Engine Division, Field Aircraft Services Ltd., Croydon.

C. J. Rogers (*Graduate*), formerly Development Engineer with Sir George Godfrey & Partners Ltd., has been appointed a Sales Engineer with the Lockheed Hydraulic Brake Co. Ltd. (Filter Division), Leamington Spa.

J. C. K. Shipp (*Associate Fellow*) has been appointed Aviation Representative (North America) for the English Electric Group.

P. G. Ware (*Associate*), formerly Chief Experimental Engineer with C.A.V. Ltd., has joined Dowty Fuel Systems Ltd. as Commercial Manager.

Graduates' and Students' Section
Friday 30th October—Reception

The Annual Reception and Dance of the Graduates' and Students' Section will be held at 4 Hamilton Place, London, W.1, on Friday 30th October 1953 from 7-11 p.m. Graduates and Students are asked to note this date. Further particulars will be published in the October Journal.

Graduates' and Students' Section

A visit has been arranged to the Bankside Power Station for Tuesday 22nd September 1953, at 6.30 p.m. Applications should be sent as soon as possible to the Hon. Visits Secretary, C. B. Redgate, stating full name, *nationality*, and grade of membership of the Society.

Conference on "Problems of Aircraft Production"

The Institution of Production Engineers is holding another Conference at the University of Southampton on the 18th and 19th December 1953. Members of the Society are invited to attend and applications for tickets should be made to the Secretary, Institution of Production Engineers, 36 Portman Square, London, W.1.

The Aeronautical Quarterly

Part 3, Volume IV (August 1953) of the Aeronautical Quarterly will be available from the offices of the Society on 15th September 1953 at 7s. 9d. ($1.10), including postage, to members of the Society, or 10s. 3d. ($1.45) to non-members.

The Contents of Part 3, Vol. IV, are:—

The Arithmetic of Field Equations	*A. Thom*
On the Low Aspect Ratio Oscillating Rectangular Wing in Supersonic Flow	*J. W. Miles*
A Generalised Approach to the Local Instability of Certain Thin-Walled Struts	*A. H. Chilver*
The Distribution of Lift over the Surface of Swept Wings	*D. Küchemann*
Note on the Stabilisation of Long Struts by an Elastic Medium	*W. J. Goodey*
Supersonic Flow Past Wing-Body Combinations	*W. Chester*

Elections

The following is a list of new members and transfers of Membership of the Society:

Associate Fellows

Dudley Cemm
Reginald John Chick
Harry Clarke
Henry Clayton (*from Associate*)
John Stuart Collins (*from Graduate*)
Raymond Laurence Coombs
Albert Edward Davies
Walter Gale (*from Associate*)
Walter Frame Gibb (*from Associate*)
William Joseph Gornall (*from Student*)
Thomas Cyril Greaves (*from Graduate*)
Stuart Charles Haynes (*from Graduate*)
John Henry Holdham
Jack Eric Lauderdale (*from Associate*)
Norman Alexander Tait
Charles Victor George Usher (*from Associate*)
Adriaan Isak van de Vooren
Arthur Joseph Walls (*from Graduate*)
Leonard Sydney Whicher (*from Associate*)

Associates

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Bernard George Brown
Douglas Alfred Cartwright
David Forbes Chalmers
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Frederick James Fellows
Frank Robert Edward Flower
Charles Henry Forster
Stanley Neville Garbutt
Kenneth Parfitt Godfrey
Jean Marius Grebil
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William John Graham Hastings
Wilfred Gilbert Frank Ilott
Leonard William Kilbourne (*from Student*)
Desmond Anthony Magill
Donald McKay Ridland
Clive Westgarth Simpson (*from Student*)
Bhopindra Singh
Charles John Vincent Walters
John William Whiskin
Gordon Frederick Winham

Graduates

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Peter Henry Frederick Burton (*from Student*)
Stanley Arthur Griffin
Stanley Charles Owers
Yvonne Mary Poulter
Victor Frederic Thompson (*from Student*)
John Roger Wilkinson

Students

Ronald Coleman
Hafeez Ud Deen
John Frederick Gordon Hart
V. D. Lakhanpal
John Rendell Conrad Pedersen

Companions

Desider Golten
Norton Hume Smith

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September 14th

41st Wilbur Wright Memorial Lecture—Structures—Buckling and Instability. Professor N. J. Hoff. Royal Institution, Albemarle Street, London, W.1, at 6 p.m.

RECEPTION.—4 Hamilton Place, W.1, following the Wilbur Wright Memorial Lecture. (By ticket only.)

September 15th-17th

Fourth Anglo-American Aeronautical Conference. Friends House, Euston Road, and University College, Gower Street, W.C.1. *Delegates only, to whom an amended time-table has been sent.*

September 22nd

GRADUATES' AND STUDENTS' SECTION. Visit to Bankside Power Station. *See* separate notice.

October 6th

GRADUATES' AND STUDENTS' SECTION. Discussion on the "Design of Light Aircraft" to be opened by R. Prizeman. 4 Hamilton Place, W.1. 7.30 p.m.

October 20th

GRADUATES' AND STUDENTS' SECTION. An Introduction to Gas Turbines. D. H. Mallinson. 4 Hamilton Place, W.1. 7.30 p.m.

October 29th

MAIN SOCIETY LECTURE.—Operation of Aircraft in Low Visibility Conditions. E. S. Calvert and J. Mercer. Institution of Mechanical Engineers, Storey's Gate, S.W.1. 6 p.m.

BRANCHES

September 21st

Halton.—Films. The Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

September 26th

Henlow.—Student Members' Premium Lectures. R.A.F. Technical College, Henlow. 7.30 p.m.

September 28th

Halton.—Fluid Control Waves. S. G. Shand. The Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

October 5th

Glasgow.—Helicopter Design Problems. A. H. Yates. Royal Technical College. 7.30 p.m.

Halton.—Branch Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

Henlow.—Redux. Dr. N. A. de Bruyne. R.A.F. Technical College, Henlow. 7.30 p.m.

October 10th

Halton.—Annual General Meeting at 16.00 hours followed by Flight Testing of Flying Boats. Geoffrey Tyson.

October 12th

Henlow.—Presidential Address: Air Logistics. Air Chief Marshal Sir John Whitworth Jones. R.A.F. Technical College, Henlow. 7.30 p.m.

October 19th

Henlow.—Parachute Progress and Design. S. B. Jackson. R.A.F. Technical College, Henlow. 7.30 p.m.

GENERAL GUY DU MERLE

General Guy du Merle, *Fellow*, informs us that he is a representative of the French Air Ministry on the interim Committee of E.D.C. and not Commandant des Ecoles et Stages at the Ministry of Public Works and Transport, as stated in the introduction to the Sixth British Commonwealth and Empire Lecture in the July 1953 JOURNAL.

B.S.I.—CHANGE OF ADDRESS

Members are asked to note that the British Standards Institution is now occupying its new premises at 2 Park Street, London, W.1. (Telephone: Mayfair 9000.)

JOURNAL BINDING

Self-Binder Cases

Self-Binder cases of the "Easibind" type are available from the offices of the Society. These binders are for members who do not have their Journals permanently bound, or who wish to keep their Journals together during the year for binding later.

These cases will hold 12 Journals which are kept in place by means of flexible steel wires. Journals can be inserted or withdrawn easily without damage, so preserving the contents for permanent binding later. The Journal will open flat at any page.

The binder is strongly made in durable dark blue leather cloth on stiff board covers and has gold lettering on the spine. The year is not blocked on the spine but there is a panel on which members who wish to use the binder as a permanent case can put the date.

The cost is 11s. 6d. each including postage and packing for either the size to fit 1952 and previous Journals, or for the size to fit the Journal from January 1953, which has been increased in size. Orders and remittances should be sent direct to the Secretary at the Offices of the Society and it is *important to state whether the old size or new size is required.*

Permanent Binding

There is no increase in the price of permanent binding of Journals. The prices are:—

1952 Volume (including packing and postage) 16s. 0d.
Previous Volumes (including packing and postage) 18s. 0d.

Journals, *with a note of the name and address of the sender,* should be sent direct to the Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month in order to be effective for the* JOURNAL for the following month.

The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 57 SEPTEMBER 1953 NUMBER 513

Civil Jet Operations

by

CAPTAIN A. M. A. MAJENDIE, M.A., A.F.R.Ae.S.

(Flight Captain, Comet Fleet, British Overseas Airways Corporation)

The 882nd Lecture to be given before the Royal Aeronautical Society was held under the auspices of the Glasgow Branch on 23rd April 1953. Professor W. J. Duncan, F.R.S., F.R.Ae.S., President of the Glasgow Branch, welcomed the visitors. He expressed regret that the President of the Society, Mr. G. H. Dowty, F.R.Ae.S., was unable to be present and welcomed Mr. N. E. Rowe, C.B.E., F.R.Ae.S., Chairman of the Branches Committee, and the Secretary of the Society, Dr. A. M. Ballantyne, T.D., A.F.R.Ae.S.

Mr. Rowe, taking the Chair for the rest of the meeting, said how much the President regretted not being present; as they knew, he had shown the liveliest interest in the Branches and had made many visits to them during his year of office. This was the twelfth Main Society Lecture to be given at a Branch meeting; the holding of Main Lectures at the Branches had proved a great success and he was sure that the present meeting would be a success also, because their lecturer had been doing pioneering work for the past three years on the development of the Comet. After graduating from Cambridge in 1940 Captain Majendie joined the R.A.F. and served as an instructor for several years; he served with Coastal Command in 1943 and was mentioned in despatches, then served on ferry duties, and in 1946 joined British Overseas Airways. He held a Pilot's " B " Licence, a Navigator's Licence and a Wireless Operator's Licence. Before being appointed to work on the Comet he had been engaged in helping Mr. E. S. Calvert at the Royal Aircraft Establishment to develop the high intensity lighting system. Captain Majendie was one of those rare people who combined a scientific and a really practical ability.

1. Introduction

This paper is confined strictly to the operational experience gained up to the present time by The British Overseas Airways Corporation with the de Havilland Comet I, powered by four Ghost 50, centrifugal compressor type, jet engines. The subject matter is already too large to permit any incursion into the related fields of economics, or maintenance, about which the author, in any case, is inadequately qualified. Emphasis has been placed on those aspects of the operation where new ground has had to be broken, and the basic philosophy underlying the current B.O.A.C. cruising procedures for the Comet has been dealt with fairly fully on this account. Much of the material presented in this paper is not new, but, as far as is known, no previous attempt has been made to gather it all together, and this has been the main purpose in the presentation.

The author would like to take this opportunity of paying tribute to his colleagues in B.O.A.C., and to many members of the staff of the de Havilland Companies, and to many other members of the Industry, all too numerous to mention, but without whose assistance this paper could never have been written, and in particular to Kelvin & Hughes Ltd. for providing the photographs for Figs. 18 and 19. He would also like to acknowledge the formal permission of B.O.A.C. to present the paper.

Symbols and Abbreviations

V_{md}	minimum drag speed
W	weight
E.A.S.	Equivalent Air Speed
T.A.S.	True Air Speed

NOTE: One kilogram = 2·205 lb.

2. Cruising

2.1. THEORETICAL TREATMENT

The characteristics of a turbo-jet aircraft are peculiar, and bear little resemblance from an operational point of view to those of piston-engined propeller aircraft. In particular, the dictates of efficiency allow very little basic freedom in the selection of cruising procedures, although a fair latitude exists in their detailed application. The general requirements are now fairly well known, and there is no need to do more than mention the need to fly high for long-range high-speed operation with a jet aircraft. This follows from the engine requirement of high r.p.m. in order to achieve a good specific consumption; the only method of equating thrust output against airframe drag for aerodynamic efficiency then being to use altitude in place of a throttle control. By this means it is possible to select the correct lift/drag ratio for optimum range at the same time as the best engine r.p.m. for specific consumption, equating

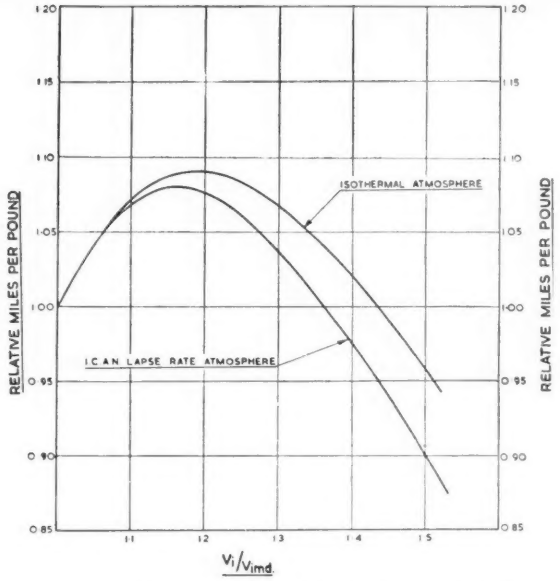

FIGURE 1. Specific range—Isothermal and lapse rate conditions.

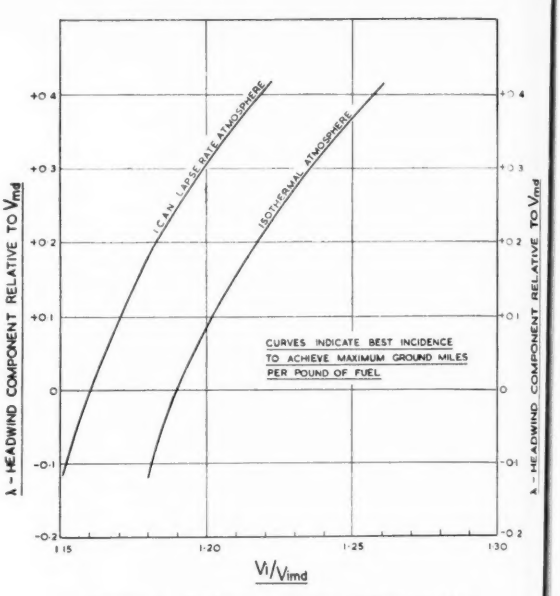
λ - HEADWIND COMPONENT RELATIVE TO Vmd
I.C.A.N. LAPSE RATE ATMOSPHERE
ISOTHERMAL ATMOSPHERE
CURVES INDICATE BEST INCIDENCE TO ACHIEVE MAXIMUM GROUND MILES PER POUND OF FUEL
Vi/Vimd

FIGURE 2. Optimum cruising—Effect of wind.

thrust output with drag by flying at the correct altitude to achieve this for any given aircraft weight.

In practice with a centrifugal compressor type engine, best engine specific consumption (lbs. fuel/lb. thrust) per hour, is generally obtained at the maximum permitted r.p.m. consistent with a satisfactory overhaul life. For the purpose of a simple analysis the engine r.p.m. therefore can be assumed constant, variations of thrust output with altitude being dependent only on air density to a first order of accuracy. Because of the high fuel consumption of jet engines, in comparison with piston ones, a relatively greater percentage of the aircraft all-up weight is burnt as fuel each hour of flight, and the total fuel load at take-off can approach 50 per cent. of the total weight. It follows that there is a large progressive reduction of all-up weight throughout the flight, and that the thrust requirements demanded of the engine for a given lift/drag ratio decrease in the same proportion. To achieve this at constant r.p.m., height has to be increased progressively as weight diminishes, and this leads to the now well-known cruising-climb technique.

Under level flight conditions, if engine thrust can be varied without change of specific consumption, it can be shown that best range will be obtained by flying at the lift/drag ratio appropriate to a speed a little more than 30 per cent. above that for minimum drag, as long as this speed is below the drag critical Mach number. In practice, for level flight operation at low altitude, considerable engine throttling is needed to achieve this speed. As engine specific consumption worsens with reduction in r.p.m. it is advantageous on multi-engined aircraft to derive the greater part of the total required thrust from one or two engines operating near optimum r.p.m., and to idle or shut down the remaining engines. The same argument holds good when holding at minimum consumption (*i.e.* maximum endurance conditions), which is achieved by flying as close to the speed for minimum drag as possible, consistent with altitude stability.

When engine r.p.m. is held constant at optimum engine conditions, any required lift/drag ratio can be selected by flying at the equivalent air speed appropriate to it for the aircraft weight at that time. It is convenient to relate this air speed to that for minimum drag, as by so doing it can be expressed independently of aircraft weight for a given lift/drag ratio or lift coefficient. Under these conditions of set engine r.p.m. and selected equivalent air speed at a given percentage above that for minimum drag, thrust required varies only with weight, and cruising height will settle down with the engine altitude-throttled to give the required figure, height increasing as fuel is consumed and weight diminishes. Operating in this way best range conditions are achieved at a rather slower air speed than for the level flight case already discussed, as true air speed does not fall as fast as equivalent air speed when the latter is reduced, owing to the increase in cruising height associated with such a reduction. Furthermore, below the tropopause, the temperature lapse rate gives an improvement in engine performance, over isothermal conditions, for a reduction of equivalent air speed, and consequent increase in cruising height, making an even slower operation desirable in this case.

These effects are illustrated in Fig. 1, which has been drawn from a simplified analysis for operation below the drag critical Mach number. It will be seen that best range occurs at about 1·19 V_{md} under isothermal conditions, and at about 1·16 V_{md} under standard temperature lapse rate conditions. These curves are for still air conditions, and the effect of a wind component on best range cruising speed is shown in Fig. 2, the wind component being shown as a fraction of V_{md}. Figs. 3 and 4 show the effect of wind and temperature on range, based on this simplified analysis.

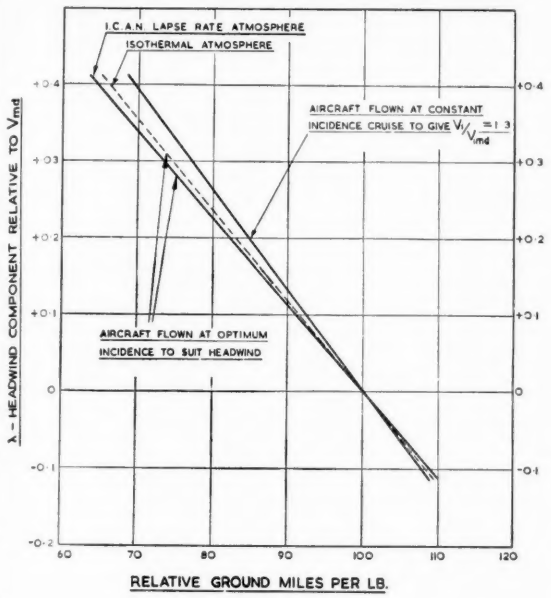
I.C.A.N. LAPSE RATE ATMOSPHERE
ISOTHERMAL ATMOSPHERE
AIRCRAFT FLOWN AT CONSTANT INCIDENCE CRUISE TO GIVE $V_i/V_{md} = 1.3$
AIRCRAFT FLOWN AT OPTIMUM INCIDENCE TO SUIT HEADWIND
λ - HEADWIND COMPONENT RELATIVE TO V_{md}
RELATIVE GROUND MILES PER LB.

FIGURE 3. Effect of wind on range.

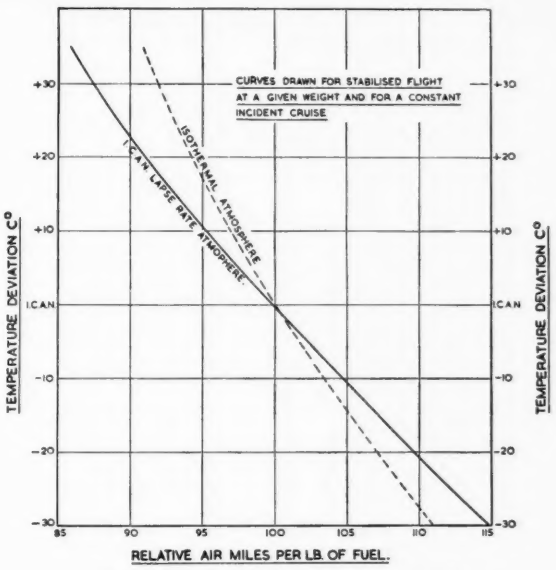
CURVES DRAWN FOR STABILISED FLIGHT AT A GIVEN WEIGHT AND FOR A CONSTANT INCIDENT CRUISE
ISOTHERMAL ATMOSPHERE
I.C.A.N. LAPSE RATE ATMOSPHERE
TEMPERATURE DEVIATION C°
RELATIVE AIR MILES PER LB. OF FUEL.

FIGURE 4. Effect of temperature on range.

It might be thought that a limitation on the selection of a cruising equivalent air speed would be imposed on grounds of stability, because of the need to preserve a suitable margin above V_{md}. It will be realised that this is not so since no attempt is being made to fly at a set altitude, and any temporary excess of drag over thrust will result in a descent to a level at which adequate engine output is again available. There is no theoretical or practical objection to a cruising-climb at, or near, V_{md} on this account, as long as other limitations do not intervene. This effect is shown in Fig. 5.

2.2. REQUIREMENTS OF A PRACTICAL CRUISE CONTROL PROCEDURE

To carry out an actual operation successfully and efficiently, it is necessary to devise a practical cruise control procedure, so that the operation can be kept within practical limits of the optimum condition, without placing an undue burden upon the operating crew, and within the limits of available instrumentation. The acceptable tolerances in this connection are much smaller for a jet than for a piston-engined aircraft, owing to the much greater weight of fuel burnt every hour expressed as a percentage of available payload capacity. As a result, relatively small increases in specific range (air miles/lb. fuel) can improve considerably the payload capacity under limiting conditions.

Apart from being workable and efficient, the cruise control procedure adopted must enable a satisfactory presentation to be made of all those operational factors needed in the air and for flight planning. These include true air speed, time, distance, fuel, weight, height, endurance and so on, for an operation under given conditions. A vital factor in air safety is the presentation of this information to the operating crew in a simple and unambiguous manner, particularly for jet operations when time itself is at a premium.

2.3. MEASURED PERFORMANCE

A typical chart of specific range against indicated air speed is shown in Fig. 6, which has been drawn for temperature conditions 15 C. Degs. above standard, assuming a continuing lapse rate and no tropopause. For practical purposes it is convenient to suppress the tropopause in this way and so to avoid discontinuities in presentation, particularly as most current operations are conducted in the troposphere. It will be seen from Fig. 6 that the variation of specific range with indicated air speed for a given weight is not as critical as might be expected theoretically; nevertheless, it is as well to stress again the importance of small changes in specific range when expressed in terms of payload. It will be

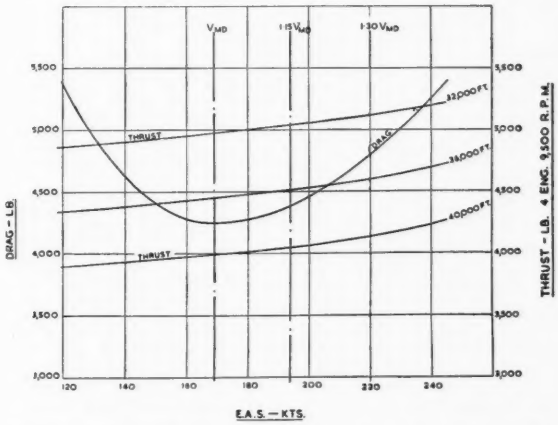

FIGURE 5. Stability at near V_{md} operation: typical drag and thrust variation with E.A.S.

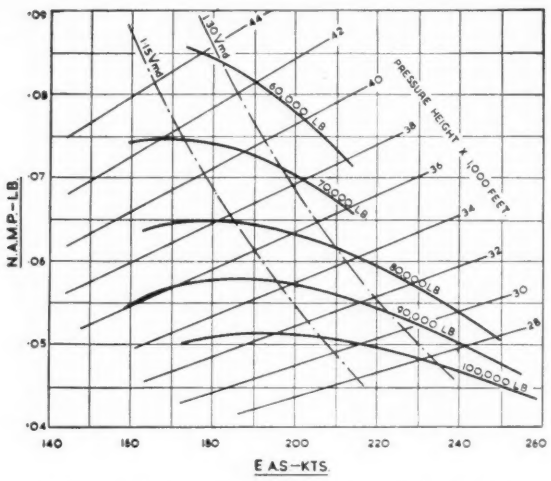

FIGURE 6. Typical performance curves based on E.A.S. 9,500 r.p.m., +15 Deg. C. above standard temperatures.

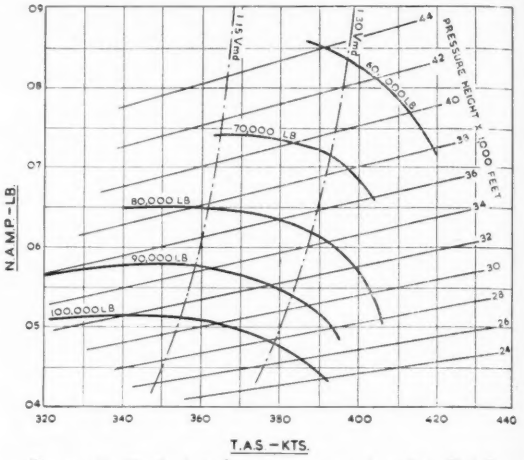

FIGURE 7. Typical performance curves based on T.A.S 9,500 r.p.m., +15 Deg. C. above standard temperatures.

appreciated, however, that quite a lot of flexibility exists in the choice of a satisfactory procedure.

It must also be remembered that the choice of a cruising procedure cannot be divorced from the climb fuel needed to reach its starting point. A small gain in specific range at altitude on a short sector can be completely discounted by the extra fuel needed to climb to a higher initial cruising level. To some extent the optimum procedure has consequently to be related to stage length.

Figure 7 shows specific range against true air speed for similar temperature conditions. It will be seen that there is a very heavy fuel penalty if an operation is attempted away from the best range conditions, to gain a worth while increase in speed.

On current operations with the Comet I, B.O.A.C. is using a constant incidence cruise procedure, achieved by flying set values of indicated air speed against aircraft weight, an adjustment being made every half hour. For a given incidence and set engine r.p.m. it is possible to consolidate all the cruising information onto a single graph, as shown in Fig. 8. From this it can be seen that variations in temperature have surprisingly little effect upon cruising endurance; they do, however, have a considerable effect upon cruising height. It might be supposed from this that flight under conditions of pronounced horizontal temperature gradient, causing rapid changes in cruising level, would show marked deviations in performance from normal conditions. So far it has not been possible to detect any such effect.

Under constant incidence cruise conditions at set engine r.p.m. it can be shown quite simply that both true air speed and Mach number are uniquely related to ambient temperature only, irrespective of aircraft weight or cruising height, provided that the flight is under stabilised conditions. These relationships enable useful simplifications to be made in the tabulation of cruise control data. It should be noted that Mach number increases as ambient temperature falls, and that operation below a given temperature may lead either to an excessive drag rise, or to control difficulties. Such a condition is not encountered on the Comet I, but if it were it would then probably be best to abandon the constant incidence technique upon reaching a suitable Mach number, thereafter operating at this constant Mach number.

2.4. CRUISE CONTROL IN PRACTICE

Although it is possible to conduct an operation from a suitable set of performance graphs, similar to the figures already shown, this does not provide a very good method of presentation for practical use. Experience has shown that tabular information is less likely to be misinterpreted, and is generally easier to use, than are graphs. Figs. 9(*a*) and 9(*b*) are examples of current B.O.A.C. tables for a 1·3 V_{md} cruise, the one normally used, comprising between them a complete statement of all relevant information needed for a given cruising procedure. There is not space in this paper to give a complete account of the procedures for which these tables have been designed, but their general form is self-explanatory.

The effect of a wind component upon the cruising procedure is shown in Fig. 10. Even a headwind component of the order of 100 knots makes very little difference to the optimum cruising conditions and, as far as Comet I operations are concerned, it is current practice not to alter the standard procedure even when extreme conditions are encountered. It might be expected that there would be a potential gain in cruising at a different level under strong wind conditions, particularly if the wind were rather localised, as in a jet stream. This is not so as this type of aircraft is particularly inflexible as far as altitude is concerned. The normal cruising level is very close to the absolute ceiling, precluding any attempt to climb out of strong wind conditions, and the range penalty for descending is so heavy that the headwind component would have to fall off by considerably more than 10 knots per thousand ft. loss of height to justify descending. The only correct technique to avoid jet stream winds is by flying out of their effect horizontally.

PRESS HT. FT. X 1,000	STAND TEMP. DEG. C
0	15
2	11
4	7
6	3
8	−1
10	−5
12	−9
14	−13
16	−17
18	−21
20	−25
22	−29
24	−33
26	−37
28	−41
30	−45
32	−49
34	−53
36	−57
38	−61
40	−65
42	−69
44	−73

WEIGHT LB. X 1,000	I.A.S.-KNOTS
100	240
95	234
90	228
85	222
80	216
75	210
70	203
65	196
60	189

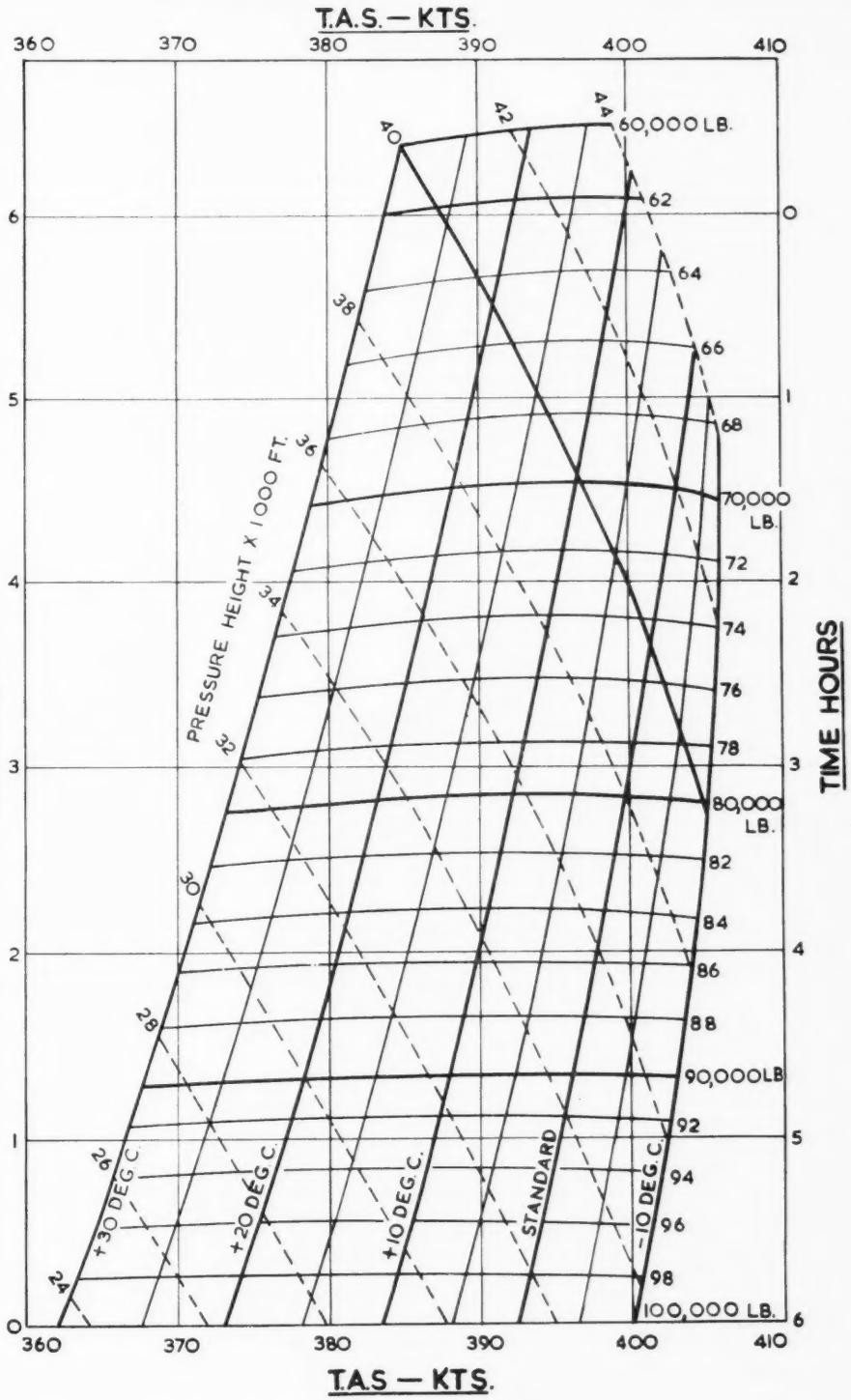

FIGURE 8. Generalised presentation for a given operation (4 engines—9,500 r.p.m.—1·30 V_{md}).

Time on cruise hrs.	*Wt. kgs.* × 1,000	*Cruising air speed (T.A.S. kts.) versus Temperature deviation from standard*									*Wt. kgs.* × 1,000	*Time on cruise hrs.*
		−10	−5	0	+5	+10	+15	+20	+25	+30		
0	47·0	402	399	395	391	387	382	377	372	367	47·0	0
1	44·0	403	400	397	393	389	385	380	375	370	44·0	1
2	41·0	405	402	399	396	392	388	383	378	373	41·0	2
3	38·0	406	404	401	398	394	390	386	382	377	38·0	3
4	35·0	407	406	403	400	397	393	389	385	380	35·0	4
5	32·0	407	407	405	402	399	396	392	388	384	32·0	5
6	29·0	407	407	407	405	402	399	395	392	388	29·0	6

B.O.A.C. O.D.U. COMET I (4 Ghost 50)
CRUISE TABLE
4 eng., 9,500 r.p.m., 1·30 V_{md}.

Sector distance (n.mls.)	500	1,000	1,500
Additional fuel (kgs.)	1,700	1,500	1,300
Additional time (mins.)	15	15	15

Hours	*Aircraft weight (kgs. × 1,000) versus Time on cruise (hrs. mins.)*																*Hours*
	0	1	2			3			4			5			6			
temp. / *mins.*	−10 *to* +30	−10 *to* +30	−10 *to* +4	+5 *to* +20	+21 *to* +30	−10 *to* +4	+5 *to* +20	+21 *to* +30	−10 *to* +4	+5 *to* +20	+21 *to* +30	−10 *to* +4	+5 *to* +20	+21 *to* +30	−10 *to* +4	+5 *to* +20	+21 *to* +30	*temp.* / *mins.*
0	47·65	43·95	40·45	40·45	40·45	37·4	37·4	37·25	34·45	34·5	34·3	31·75	31·8	31·65	29·2	29·3	29·15	0
2	47·5	43·8	40·5	40·4	40·30	37·3	37·3	37·15	34·35	34·4	34·2	31·65	31·75	31·6	29·1	29·25	29·1	2
4	47·4	43·7	40·3	40·3	40·15	37·2	37·2	37·05	34·25	34·3	34·15	31·55	31·65	31·5	29·05	29·15	29·0	4
6	47·25	43·55	40·2	40·2	40·05	37·1	37·1	36·95	34·2	34·25	34·05	31·5	31·55	31·4	28·95	29·05	28·95	6
8	47·15	43·45	40·1	40·1	39·95	37·0	37·0	36·85	34·1	34·15	33·95	31·4	31·45	31·3	28·85	29·0	28·85	8
10	47·0	43·35	39·95	39·95	39·85	36·9	36·9	36·75	34·0	34·05	33·85	31·3	31·4	31·25	28·8	28·9	28·75	10
12	46·85	43·2	39·85	39·85	39·75	36·8	36·8	36·65	33·9	33·95	33·75	31·2	31·3	31·15	28·7	28·85	28·7	12
14	46·75	43·1	39·75	39·75	39·6	36·7	36·7	36·5	33·8	33·85	33·7	31·15	31·25	31·05	28·65	28·75	28·6	14
16	46·6	43·0	39·65	39·65	39·5	36·6	36·6	36·4	33·7	33·75	33·6	31·05	31·15	31·0	28·55	28·65	28·55	16
18	46·5	42·85	39·55	39·55	39·4	36·5	36·5	36·3	33·6	33·65	33·5	30·95	31·05	30·9	28·45	28·6	28·45	18
20	46·35	42·75	39·45	39·45	39·3	36·4	36·4	36·2	33·55	33·6	33·4	30·85	31·0	30·8	28·4	28·55	28·4	20
22	46·25	42·65	39·35	39·35	39·2	36·3	36·3	36·1	33·45	33·5	33·35	30·8	30·9	30·75	28·3	28·45	28·3	22
24	46·1	42·5	39·25	39·25	39·1	36·2	36·2	36·0	33·35	33·4	33·25	30·7	30·8	30·65	28·25	28·35	28·25	24
26	45·95	42·4	39·15	39·15	39·0	36·1	36·1	35·9	33·25	33·3	33·15	30·6	30·7	30·55	28·15	28·3	28·15	26
28	45·85	42·3	39·05	39·05	38·9	36·0	36·0	35·8	33·15	33·25	33·05	30·55	30·65	30·5	28·1	28·2	28·1	28
30	45·7	42·2	38·9	38·9	38·8	35·9	35·9	35·7	33·1	33·15	33·0	30·45	30·55	30·4	28·0	28·15	28·05	30
32	45·6	42·05	38·8	38·8	38·7	35·85	35·85	35·65	33·0	33·05	32·9	30·35	30·45	30·3	27·95	28·05	27·95	32
34	45·5	41·95	38·7	38·7	38·55	35·75	35·75	35·55	32·9	32·95	32·8	30·25	30·35	30·25	27·85	28·0	27·9	34
36	45·35	41·85	38·6	38·6	38·45	35·65	35·65	35·45	32·8	32·85	32·7	30·2	30·3	30·15	27·8	27·9	27·8	36
38	45·25	41·7	38·5	38·5	38·35	35·55	35·55	35·35	32·75	32·8	32·65	30·1	30·2	30·05	27·7	27·85	27·75	38
40	45·1	41·6	38·4	38·4	38·25	35·45	35·45	35·25	32·65	32·7	32·55	30·0	30·15	30·0	27·6	27·8	27·65	40
42	45·0	41·5	38·3	38·3	38·15	35·35	35·35	35·15	32·55	32·6	32·45	29·95	30·05	29·9				42
44	44·85	41·4	38·2	38·2	38·05	35·25	35·25	35·05	32·45	32·5	32·35	29·85	29·95	29·8				44
46	44·75	41·3	38·1	38·1	37·95	35·15	35·15	34·95	32·35	32·45	32·25	29·75	29·85	29·75				46
48	44·65	41·15	38·0	38·0	37·85	35·05	35·05	34·85	32·25	32·35	32·2	29·7	29·8	29·65				48
50	44·5	41·05	37·9	37·9	37·75	34·95	35·0	34·75	32·2	32·25	32·1	29·6	29·7	29·55				50
52	44·4	40·95	37·8	37·8	37·65	34·85	34·9	34·65	32·1	32·15	32·0	29·55	29·65	29·5				52
54	44·25	40·85	37·7	37·7	37·55	34·75	34·8	34·6	32·0	32·1	31·95	29·45	29·55	29·4				54
56	44·15	40·75	37·6	37·6	37·45	34·65	34·7	34·5	31·9	32·0	31·85	29·35	29·45	29·35				56
58	44·05	40·6	37·5	37·5	37·35	34·55	34·6	34·4	31·85	31·9	31·75	29·25	29·4	29·25				58
60	43·95	40·45	37·4	37·4	37·25	34·45	34·5	34·3	31·75	31·8	31·65	29·2	29·3	29·15				60

Amb. air temp. °C.	*T. A. S. kts.*
−70	405
−65	404
−60	402
−55	399
−50	397
−45	394
−40	392
−35	389
−30	386
−25	383
−20	380
−15	376
−10	373
− 5	369

FIGURE 9(*a*). Tabular presentation—Cruise table.

| | | |

2.5. ENGINE FAILURE

Before leaving the subject of cruising some reference must be made to the subject of engine failure. If an engine fails at cruising height the aircraft will then be above both its three-engined ceiling and the optimum level for a three-engined cruise, which for the Comet is about 10,000 ft. lower, for a given weight. Although specific range under stabilised three-engined cruise conditions is seriously below that for a normal operation, being about 20 per cent. less, the benefit of the excess height at engine failure considerably reduces the penalty. In order to obtain the maximum benefit from the extra height, a carefully controlled descent to the three-engined cruise level is made, taking about an hour; for ease of reference this controlled descent is now known as a "drift-down."

3. Climb and Descent

A much larger proportion of a normal flight is devoted to climbing up to the cruising level and descending from it than is normally the case with a piston-engined propeller aircraft. The operation of a jet aircraft bears a much closer resemblance to the flight of a projectile than does that of traditional types. For this reason it will be seen that overall efficiency can be considerably affected by using different techniques for climb or descent. As specific range is extremely poor at low altitudes, there is an obvious requirement to climb as fast as possible directly after take-off, and maximum permissible engine r.p.m. is used for this purpose. Considerable latitude exists in the choice of climbing speed as far as performance is concerned; a fast air speed on the climb being theoretically better than a slow one at any given point for specific range at a reasonable rate of climb, but demanding a prolonged period of flattened climb at low level, and excessive engine consumption, in order to gain the initial speed. Such a climb also suffers from the disadvantage of possibly providing an operation at speeds well above those recommended for use under turbulent conditions, which are not infrequently encountered in the earlier stages of climb after take-off.

As far as descent is concerned, the most efficient method is a long shallow operation, which, on range alone, provides a gain of about 20 per cent. over normal cruising for an equivalent distance. However, if height is held until nearly overhead destination, and a rapid descent is then made, there is a penalty in view of the fuel burnt during this rapid descent, it being impracticable to shut down the engines owing to the requirements of pressurisation and other ancillary power demands. This further penalty lifts the total fuel requirement for a delayed descent to a figure about 40 per cent. above that needed in the optimum case, not taking into account the maximum descent rate which can be tolerated in the passenger cabin of a public transport aircraft, which would still further increase the penalty. The importance of this limitation will be understood from the example of the Comet I, which is pressurised to a maximum working differential of $8\frac{1}{4}$ lb./in.^{2}, providing a cabin altitude of 8,000 ft. at a cruising level of 40,000 ft. It follows that the time taken to descend from 40,000 ft.

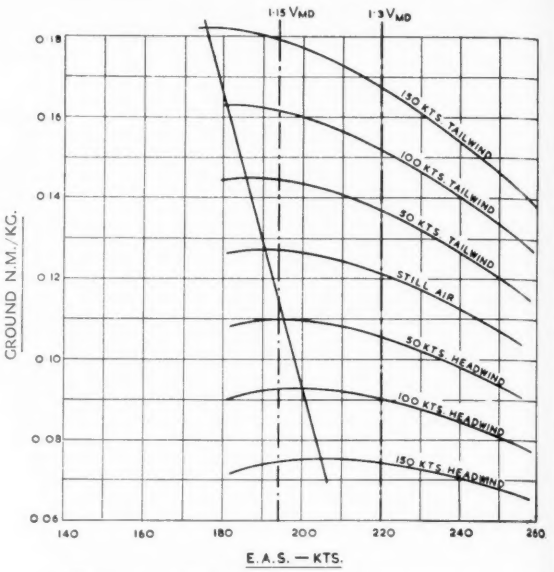

FIGURE 10. Typical effect of wind component on Ground N.M./KG.

must not be less than that needed to bring the cabin from 8,000 ft. to surface pressure. A seriously delayed descent, making allowance for this requirement, can impose an even greater penalty than the examples already given. Nevertheless, such a delayed descent has the operational advantage of preserving altitude, and therefore diversion range, to a much later stage in a given operation should weather conditions for landing be doubtful at destination. As in the case of the climb, speed on the descent is not very critical, and a fair latitude exists in making a suitable choice of procedure.

4. Take-off

Take-off technique on a civil air liner is, to a large extent, governed by the mandatory performance requirements laid down in the current Air Navigation Regulations and British Civil Airworthiness Requirements. These requirements were explained by the Chief Executive of the Air Registration Board in a recent lecture to the Society[1], which included examples of the Flight Manual presentation of the scheduled performance data for the Comet I. The purpose of this discussion will be to describe how the scheduled data is used in practice in day-to-day airline operations.

To recapitulate for the benefit of those not familiar with the current requirements, it will be remembered that take-off weight has to be restricted, if necessary, so that adequate margins of safety can be achieved as between the aircraft's performance and the available take-off run, obstacle-free distance, and subsequent climb-out. This must be so either for a normal take-off, or for one on three engines, in the event of a failure. In addition, if an engine should fail so early on the take-off run as to preclude the possibility of continuing on three engines, there must be an adequate emergency distance ahead in which the pilot can bring the aircraft safely to rest. At

all times on the take-off it must be safely possible in the event of an engine failing, either to continue or to bring the aircraft to rest.

For a given take-off on a particular runway under known conditions of wind and temperature, etc., it is possible to construct a graph, similar to that shown in Fig. 11, from the scheduled performance data in the Flight Manual. Such a graph is essentially a plot of weight against speed, although it is convenient to plot speed as the fraction of the take-off safety speed at which engine failure is assumed to occur; take-off safety speed being the minimum safe speed at which the aircraft should be climbed away from the ground, currently set at 15 per cent. above the stall. It has been suggested that this is not an adequate margin for a jet aircraft, because of the absence of slipstream over the wings and so on; experience on the Comet so far does not, however, indicate any need to increase this figure, particularly in view of the excellent asymmetric handling qualities of the aircraft due to the close spacing of the engines from the fuselage centre line, and the absence of airscrew drag from a failed engine.

Figure 11 shows that as the speed at which engine failure occurs is increased, so the requirements of take-off distance and take-off run can be met at increasing weight, while that for emergency distance can only be met by decreasing it. The kink in the line for take-off run indicates the power failure speed ratio above which the all-engines-operating requirement becomes more severe than the engine failure case. It will also be noticed that there may be a kinetic energy limit on the speed at which an emergency stop can be attempted, if there is a practical limit upon the thermal capacity of the wheel brakes.

It will be seen that the lowest point of intersection between the two sets of conflicting requirements provides the maximum weight at which all can be met in full throughout the take-off. This point, *A* in Fig. 11, also indicates the critical speed below which, if an engine should fail, an emergency stop should be made, and above which the take-off should be continued on three engines. At any lighter weight, W_1 for example, there will be a range of speeds, between *B* and *C*, throughout which either decision can be safely made; at a heavier weight, W_2 for example, there will be a range of speeds, between *D* and *E*, between which a safe operation is not possible in the event of engine failure.

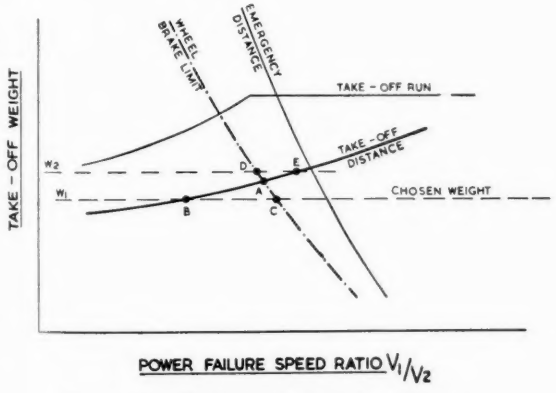

FIGURE 11. Flight manual take-off calculations.

From graphs such as Fig. 11 it is possible to construct consolidated and simplified charts for use on any given runway, an example being given in Fig. 12. For given conditions of surface wind and temperature it is then possible to read off directly the maximum permitted take-off weight, and the critical, unstick, and climb-away speeds. To simplify the use of such charts in day-to-day operations, it has become current practice in B.O.A.C. to quote only the critical speed for maximum permitted take-off weight, even when operating lighter than this. Inspection of Fig. 11 will show that this must always provide a safe criterion in the event of engine failure.

5. Landing

Experience with the Comet has shown that the certification performance requirements for landing are much less severe operationally than are those for take-off. That this is so is a measure of the success achieved by the designers of the Comet in keeping wing loadings and stalling speeds down, and in providing sufficient aerodynamic drag to compensate in large measure for the absence of propeller windmilling drag, or reverse thrust. However, because of the slow throttle response characteristics of turbine engines, and the relative aerodynamic cleanliness of any jet aircraft, rather more accurate control of both speed and height is demanded from the pilot than is generally needed on conventional types of civil aircraft. Automatic speed control on the approach would seem to be a most important field of study for future development.

Jet transport aircraft are entirely dependent on wheel brakes and nosewheel steering for final control on the landing run: asymmetric engine power for directional control, and aerodynamic reverse thrust both being noticeably absent. Although operational experience is still lacking in this matter, it would seem that there may be a major control problem when landing upon icy or slippery runways. This problem will be much accentuated in the future if wing loadings, stalling and approach speeds, and landing weight, are all allowed to increase in new types, without any offsetting improvements in methods of control on the ground. Future trends in air safety will depend in very large measure upon the extent to which it will be possible to resist the tendency to sacrifice good approach and landing qualities in the interests of high performance: there should be no place in civil aviation for " hot " aircraft.

6. Air Traffic Control

The economic penalty in delaying the approach and landing of a jet aircraft on arrival at its destination is extremely heavy, and potential diversion range is seriously reduced as altitude is lost. The general order of these effects is shown in Fig. 13, which shows a number of ways in which a given quantity of fuel can be used from a common starting point, *A*, at the top of a normal descent to destination. In this connection, it should be noted that the main penalty in low altitude

AIRCRAFT COMET

FLIGHT EASTERN

OPERATIONS MANUAL

B·O·A·C

DATE 1.11.52

2.20.1

PART II

VOLUME II

SECTION II

Replaces Sheet dated 23.10.52 (unchanged)

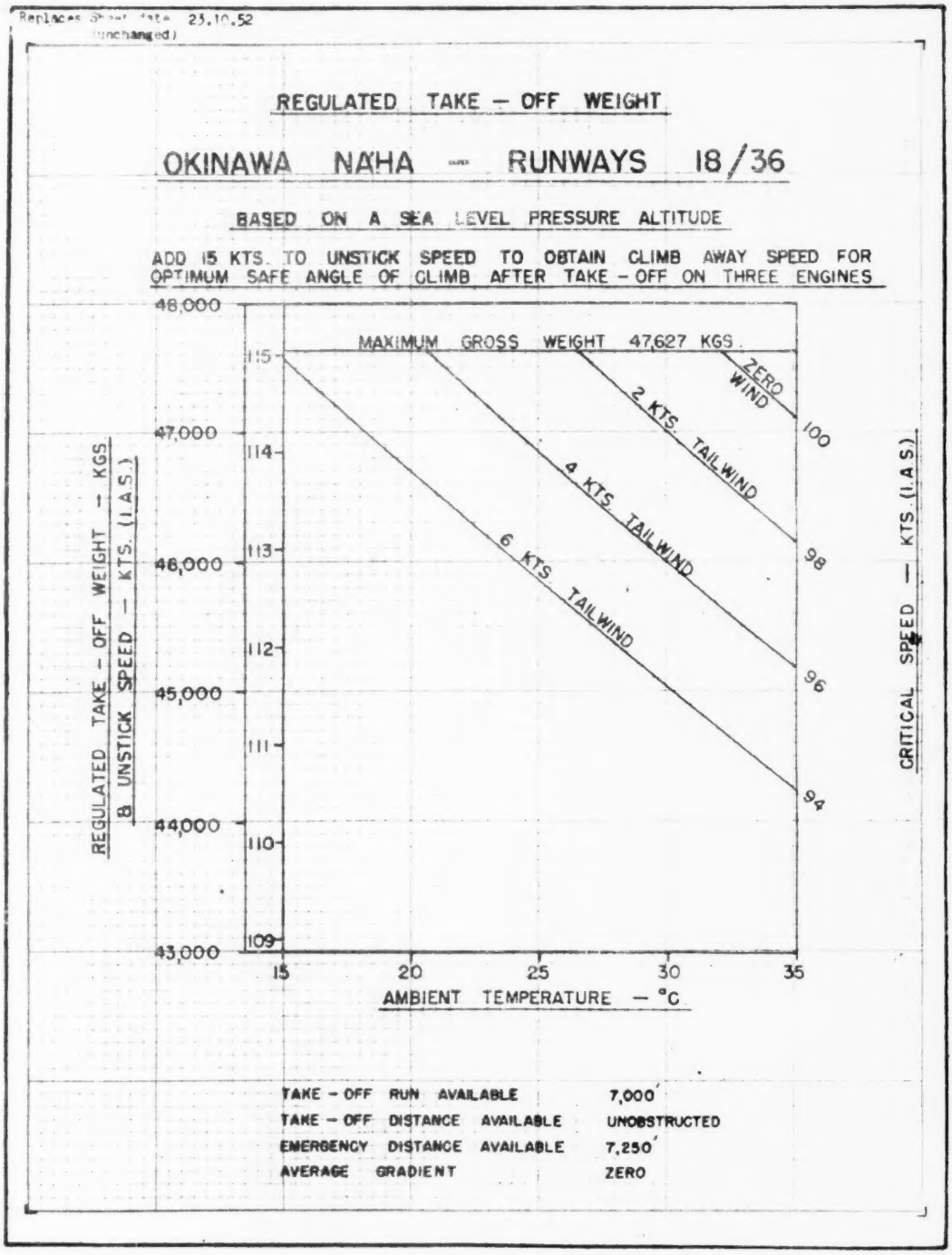

FIGURE 12. Practical take-off chart.

hol
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con
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traf

requ
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befo
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out
som
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to fo
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upon
shou
aircr
its fi
beco
along
clear

It
Utop
to de
aircra
more
than
at pr
probl
tackl

G
Come
be en
lose
flexib
cedur
exploi
unabl
in co
power

FIGUR

holding of a multi-engined jet aircraft is the reduction in available diversion range, should this be needed, and not the relatively much smaller increase in holding consumption. Similarly there is a serious range penalty if restrictive low altitude limits have to be imposed for traffic reasons upon the initial climb after take-off.

Under ideal conditions the Air Traffic Control requirements for the efficient handling of jet aircraft are, briefly, as follows. Clearance to start engines before take-off should only be given when it is possible for the aircraft to start up, taxi out, and take-off, without delay, and then to climb without restriction along some suitable routeing to its cruising altitude. At altitude, under cruising conditions, it must be cleared to follow a route which provides lateral separation from other aircraft, so that altitude restrictions are not placed upon the cruising procedure adopted. Descent clearance should not be given into a controlled air space until the aircraft can be given an uninterrupted descent path to its final approach to land. If a subsequent diversion becomes necessary, an immediate clearance to climb out along a suitable route must be given, as a low altitude clearance will penalise the operation seriously.

It will be realised that these requirements are Utopian in practice: nevertheless, it will not be possible to derive the most efficient use from civil jet transport aircraft until Air Traffic Control procedures are much more closely matched to the needs of this type of aircraft than they are now. The worst deficiencies are found at present for arrival, rather than for departure, and the problems of *en route* separation have not yet been tackled seriously at all.

Given good air brakes, such as are fitted to the Comet, descent rates of up to 4,000 ft./min. can easily be employed in instrument procedures. This ability to lose height quickly provides a very real measure of flexibility for inbound clearances, and approach procedures. This flexibility is now beginning to be exploited, and the day can be foreseen when the aircraft unable to descend steeply will be at a very serious loss in complying with Air Traffic Control requirements: powerful and effective air brakes are now becoming an essential requirement, and must be fully effective within the turbulence speed range of the aircraft and be provided with satisfactory protection under icing conditions.

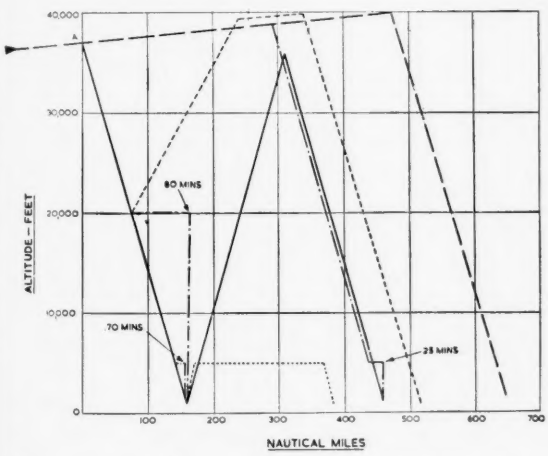


FIGURE 13. Use of reserve fuel for diversion or stand-off.

It would not be right to leave the subject of Air Traffic Control, which in any case can only be discussed here in a most superficial way, without just mentioning the associated communication problems. The very much higher speeds now being used in air transport, and the increasingly critical operational requirements, have placed new and stringent demands upon the aeronautical communication networks. These demands are not strictly speaking related solely to jet aircraft, but the introduction of the latter upon some of the world's trunk air routes has underlined glaring deficiencies in the existing arrangements. Flight safety and operational efficiency can only be built around a fast, efficient, and utterly reliable, communications network.

7. Meteorology

Some account of the meteorological aspects of civil jet operations has already been given elsewhere[2], and it is only necessary to mention one or two salient points here. When the early development route flying was started with the Comet, there was surprisingly little operational knowledge available for flight above about 20,000 ft., despite the fact that fighter aircraft had been making ascents to 40,000 ft., and above, for some years. Such ascents were strictly localised, however, and route knowledge was conspicuous by its absence.

The most important single requirement for successful jet operations is not associated with the upper cruising levels: it is the provision of really reliable forecasting of terminal and alternate weather conditions. The heavy fuel penalty of a wave-off diversion has already been stressed, and the only method of avoiding it is a really sound forecast office. At the present time it would seem to be a better investment to spend money on developing improved terminal forecasting facilities, rather than on trying to build an adequate network of upper air meteorological reporting stations. The latter project is beyond reasonable practical limits of attainment for the world's trunk air routes within the foreseeable future, while very real strides can undoubtedly be made in the provision of really accurate local forecasting. For jet operations, the aim must be to provide a service of such a standard that descent is never started towards an aerodrome, the conditions at which subsequently deteriorate to such an extent as to preclude a landing.

Despite its cruising altitude, the Comet is not yet by any means over the weather at all times. Whole stages are sometimes flown in cloud in the tropics, and severe turbulence can occasionally be encountered up to, and above, 40,000 ft. Genuine over-the-weather operation will only become possible when the cruising level is at all times in the stratosphere.

There is still a great deal to be learnt about the wind velocities aloft at the new cruising levels. Narrow belts of strong wind (rather unfortunately named "jet streams") present new problems both to the forecaster and to the pilot.

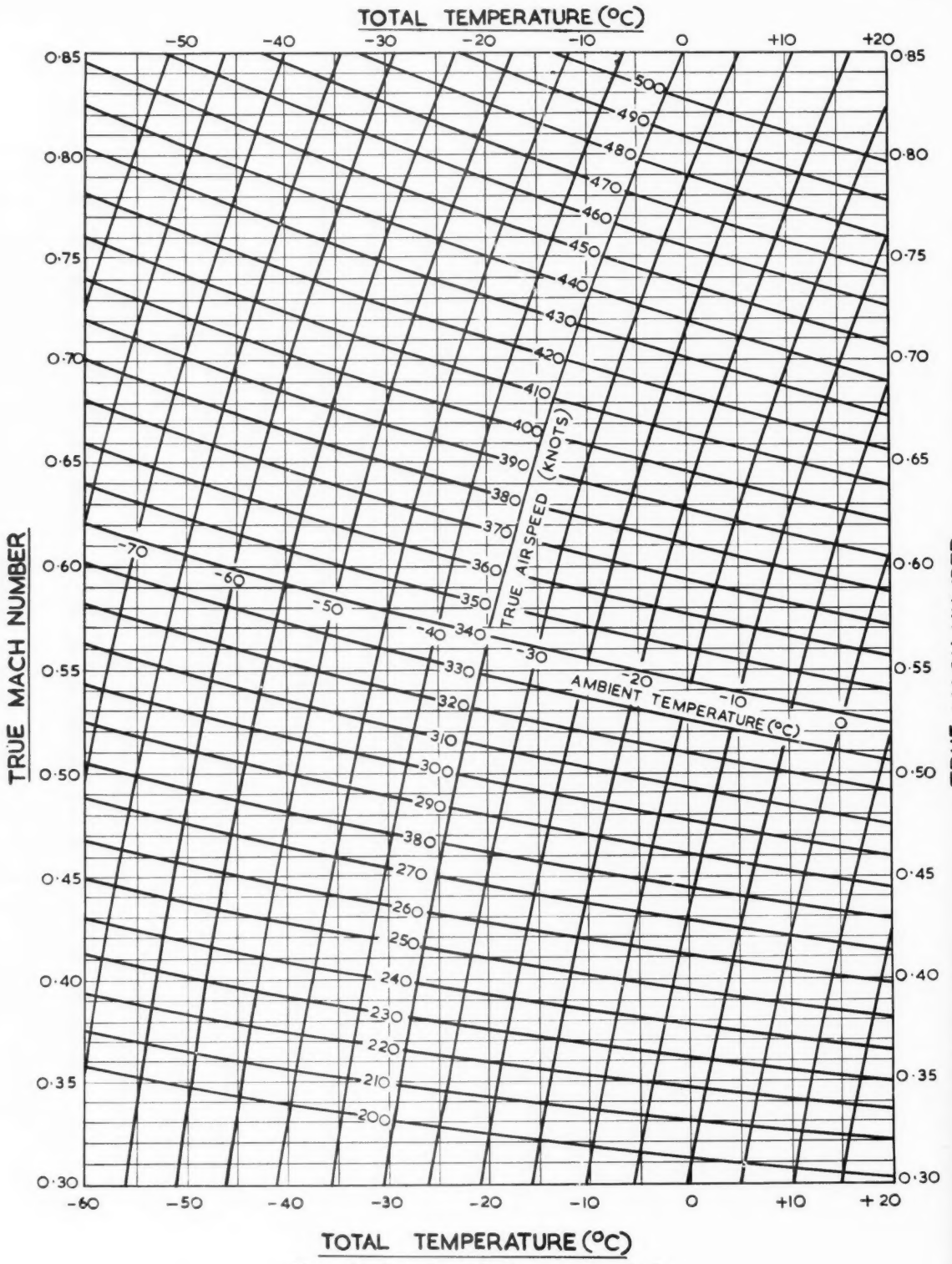


FIGURE 14. True air speed and temperature reduction.

Despite what has been said about the practical difficulty of extending adequate upper air coverage on a worldwide basis, there are undoubtedly special areas where an increase in the upper air reporting network would pay big dividends. An increase in the radio navigational cover in such areas can also play a very big part in increasing the accuracy and amount of in-flight data collected. It seems from experience gained on the Comet up to the present, that the broad picture of the upper air, already constructed on theoretical grounds, is mainly correct and that it only remains to fill in the details as operational knowledge builds up.

Before leaving this subject, the importance of the collection of meteorological statistics for planning purposes must be emphasised. This is particularly so in relation to take-off conditions from critical aerodromes, for which it is necessary to have data covering not only runway wind components and surface temperature, but also the correlation between the two. This need to relate different meteorological factors on a statistical basis is a relatively new concept, and much work of this nature will have to be done in the future.

8. Navigation

The principles of navigation of a jet aircraft are not fundamentally unlike those for any other type, but the tempo has increased considerably. Speed of working is relatively of much greater importance than absolute accuracy, and this is leading to the development of new techniques. A typical example is provided by the problem of determining true air speed. Conventionally this would be accomplished by applying to the air speed indicator reading a series of correcting factors, one of these being ambient temperature, which can itself only be determined after applying a factor dependent on true air speed to the indicated temperature. However, by combining Machmeter reading with indicated (or "gauge") temperature, it is possible to derive both true air speed and ambient temperature directly, to a reduced but acceptable order of accuracy. A chart for effecting this reduction is shown in Fig. 14: alternatively it is possible to devise a simple computor to do this.

By methods such as these, great simplifications have been introduced into the routine work of navigation. But the performance characteristics of the jet have also led to some more fundamental changes in the navigational approach. The similarity between the operation of this type of aircraft and the flight of a projectile has already been pointed out. The dependence of range and speed upon altitude now makes it impossible to present adequately the navigational situation upon a two-dimensional plotting chart: the operation has become essentially three-dimensional.

In order to achieve a satisfactory operation, it has been found necessary to establish a very close interrelationship between the cruise control and the navigation, and this has led to the need of very good teamwork between the members of the aircraft's crew. An essential requirement in the safe operation of a jet transport aircraft is a regular check upon flight progress in relation to remaining fuel. For this purpose a graph of distance to go, plotted against remaining fuel, is constructed as a standard part of the flight plan. Actual

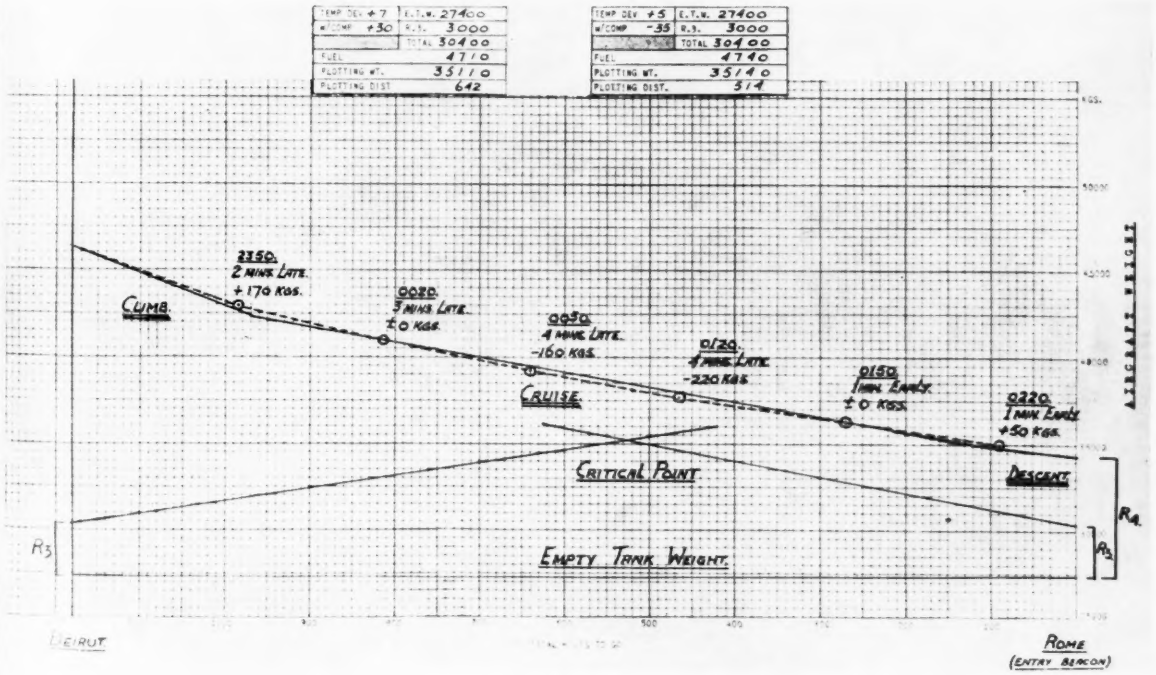


FIGURE 15. Typical "Howgozit."

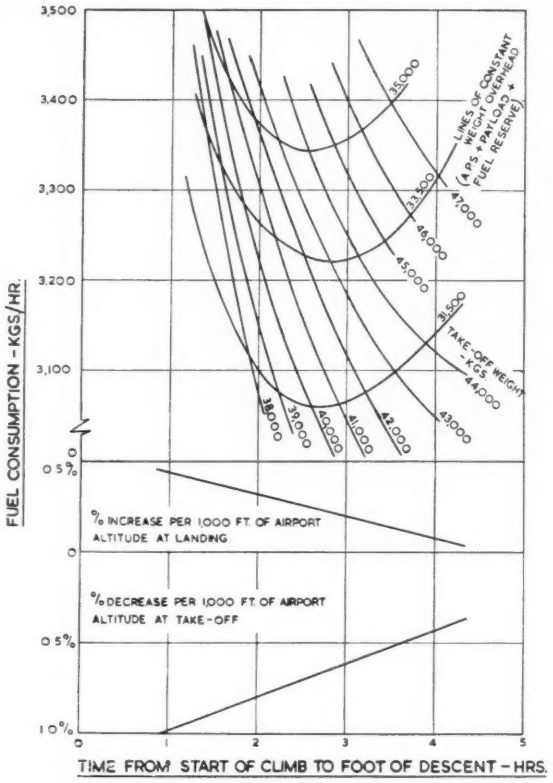


FIGURE 16. Optimum range—kg. per hour.

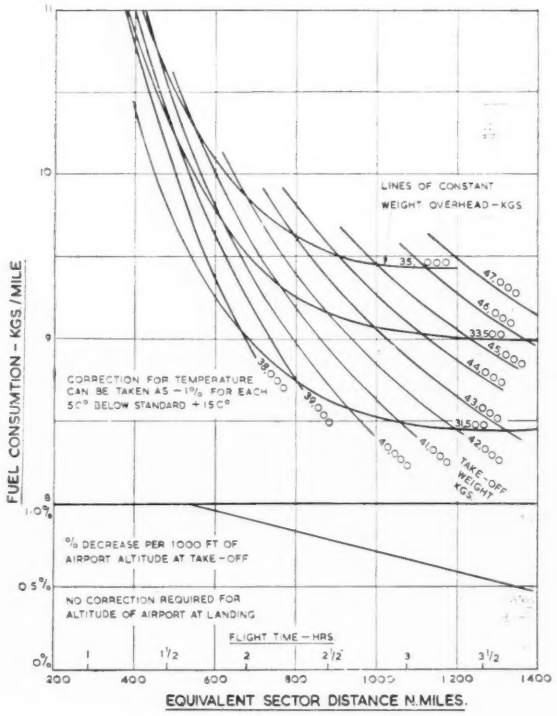


FIGURE 17 Optimum range—kg. per mile.

flight progress is then plotted on this graph, generally known as the "howgozit," as the flight proceeds, and unexpected trends become obvious very quickly. Fig. 15 is an example of an actual "howgozit," showing the manner in which the actual flight differed from the flight plan. Lines showing performance in the event of engine failure are also included on the "howgozit," and this enables a safe decision to be made without delay in an emergency.

Astronomical navigation is only used to a rather limited extent, as it is an uncertain aid dependent upon good weather conditions, and current equipment and procedures are not readily geared to the fast tempo of the operation. Surface map reading is equally doubtful and, even in good conditions, the accurate establishment of position by this means is very difficult at height. The navigation of fast, high-flying, aircraft is now essentially dependent upon radio aids. Upon the Commonwealth routes operated by B.O.A.C. these are almost exclusively confined to the medium frequency band, with all its known shortcomings. With the advent of high-speed jet operations, adequate long range track guide and distance to go information is becoming urgently needed.

9. Route Application

The civil jet air liner can only justify itself on long distance trunk route operations: it is essentially a high speed, high altitude, vehicle. The quantity of fuel needed to lift it to its normal economic cruising level is such that short-range operation cannot possibly be achieved at a reasonable mean figure for specific range. Furthermore, any increase in operating weight (*e.g.* extra fuel reserves) increases the overall consumption to a much greater extent than it does on traditional types of transport aircraft: the extra fuel burnt on an average stage being of the order of 25 per cent. of the corresponding increase in operating weight.

It follows that fuel consumed per ton mile tends to decrease as stage length, and the proportion of time at cruising altitude, both go up. Increase of stage length, however, also tends to increase the mean operating weight, and the average consumption per aircraft hour, at a given payload capacity, reaches its minimum value for a stage length well below that for best consumption per ton mile of payload carried. This effect is dependent on the fact that the most productive part of the operation is the cruise at altitude, and that short sectors, much of which are spent in climb and descent, can only be operated at a relatively low mean speed, which more than offsets any apparent benefit in reduced average hourly consumption. These general effects are shown for the Comet I in Figs. 16 and 17, which give the average consumption in kilograms per hour, and kilograms per mile, respectively, against stage length. Any failure to appreciate clearly the difference between these two quantities in assessing the potential route application of a jet transport can lead to serious errors.

10. Special Features

The advent of the jet aircraft in civil aviation has led to the introduction of a number of new features

fundamental in character, and about each of which a few comments will now be made.

10.1. POWER-OPERATED FLYING CONTROLS

The development of the modern air liner has now reached a stage at which it would be unreasonable to continue to expect the designer to achieve a satisfactory balance, over the whole available speed range, between the aerodynamic forces on the control surfaces, on the one hand, and the muscular capacity of the human pilot, on the other: power-operated controls are here to stay. Their introduction presents an opportunity for re-examining the basic requirements for safe and effective control, and this opportunity will be wasted if the primary aim in their design continues to be the emulation of previous systems. Undesirable features such as unnecessarily high break-out loads, or abnormally small control movements, must be eliminated, and aircraft response must be related to control operation in the best possible manner. This opens a whole new field of research, which we cannot afford to neglect.

10.2. AIR BRAKES

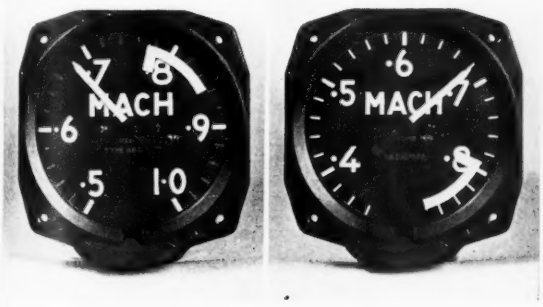
The importance of effective air brakes has already been pointed out in connection with the needs of good Air Traffic Control. Their design requirement, however, goes much farther than this, as they add a flexibility to the control of both height and speed which enables the pilot to achieve a precise operation, otherwise denied him. Additionally, they also provide a very high rate of emergency descent, should this be needed, and an ultimate method of recovering control should the pilot get into difficulties at an unduly high Mach number. Their omission from any future design would constitute a grave deficiency.

10.3. OPERATION AT RELATIVELY HIGH MACH NUMBER

Although military fighter pilots have for the past decade had to familiarise themselves with the effects of aerodynamic compressibility upon control, stability, and performance, these matters have only recently assumed any importance in civil aviation. It has already been pointed out that it is not desirable to attempt a cruise much above the drag critical Mach number, and control difficulties are not likely to be encountered until reaching some higher figure. The margin between the highest desirable cruising Mach number, and that for aerodynamic instability, or serious trim change, may not appear very large in the eyes of those accustomed to current airworthiness performance margins. Inadvertent operation to an unduly high Mach number is not likely to occur very often on civil jet aircraft, owing to the precise control of the operation called for from the pilot, and the constant attention needed to achieve this: additional safeguards in the form of audio-warning devices, etc., can also be fitted. Even if, despite all this, an unduly high figure is reached, the provision of adequate air brakes still provides an immediate solution to the pilot, should he find himself in trouble. Given adequate training, there should be no undue hazard on this account.

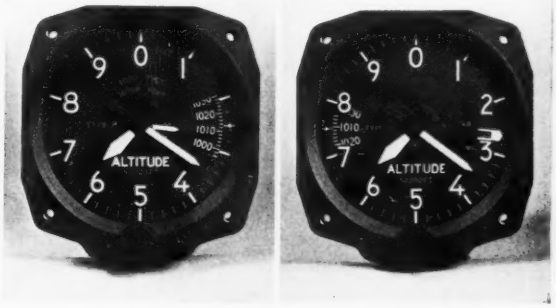
10.4. HIGH ALTITUDE PRESSURISED FLIGHT

The consequences of pressurisation failure at the new operating levels are so serious that the design of the pressure hull, and the associated method of supply and control of the pressurised air, must be beyond reasonable risk of failure. Given the necessary study, this should not be difficult to achieve. The only reasonable criticism that can be made operationally is the limitation, already referred to, in having to restrict the actual descent rate of the aircraft, because of the need to keep it in step with the cabin altitude. If the working differential pressure could be increased from 8·25 lb./in.^{2} to about 12·5 lb./in.^{2} (surely a modest increase from the engineering aspect), the cabin need never leave surface pressure, and complete operational freedom would then exist for rates of both climb and descent on this account. The provision of surface pressure in the cabin would also undoubtedly go a long way further in the elimination of discomfort and fatigue from air travel, and should be seriously considered in future design studies. Admittedly there may be too serious a structural weight penalty to allow full sea level prressure to be achieved in the cabin, but a



Old New

FIGURE 18. Old and new Machmeters.



Old New

FIGURE 19. Old and new altimeters.

Form No. 4756F. BRITISH OVERSEAS AIRWAYS CORPORATION SHEET 1

D. H. 106 COMET AIRCRAFT

FUEL/WEIGHT LOG

FUEL FLOW	2725	2730	2690	2730
R.P.M.	10250	10220	10230	10220
J.P.T.	690	680	700	680

T/O

SERV. NO. 744 / 084 A/C G-ALYP FROM LONDON

DATE 3-11-52 CAPT. JONES TO ROME

E/O GUTTRIDGE

LANDED ON CHOCKS

	START	OPEN UP	+ 3 MINS.	T.O.C					T.O.D.				
TIME G.M.T.	0929	0941	0944	0956	1008	1020	1022	1052	1116	1128	1140	1152	1166
TIME G.M.T.		TAXYING	3 MINS.	0950	1002	1014	1021	1037	1104	1122	1134	1146	TAXYING
TIME INTERVAL		:12		:12	:12	:12	:02	:30	:24	:12	:12	:12	:04
NOTE (1) FUEL FOR START & TAXY UNTIL STARTING TO OPEN THROTTLES FOR TAKE-OFF = 30 KGS/MIN.	NOTE (2) INITIAL FIGURES FROM FINAL FUEL/WT. CHIT	FUEL FLOW	1	1675	1125	875	760	750	750	600	500	800	—
			2	1650	1150	975	870	760	750	300	250	800	—
			3	1650	1150	800	720	780	750	250	200	800	—
			4	1675	1150	900	800	760	750	600	500	800	—
		TOTAL FLOW		6650	4575	3560	3150	3050	3000	1750	1450	3200	—
FLOW WEIGHT	42065	41705	41205	39875	38960	38250	38147	36422	35422	35072	34782	34142	34062
EMPTY TANK WEIGHT	28115	28115	28115	28115	28115	28115	28115	28115	28115	28115	28115	28115	28115
FUEL LEFT (F/M)	13950	13590	13090	11760	10845	10135	10032	8507	7307	6957	6667	6027	5947
INTERVAL FUEL USED		360	500	1330	915	710	103	1525	1200	350	290	640	80
TOTAL FUEL USED (F/M's)		360	860	2190	3105	3815	3918	5443	6643	6993	7283	7923	8003
FUEL CHECK LINE	13950	13950	13950	13950	13960	13950	13950	13950	13950	13950	13950	13950	13950
TANKS P.O.	4600	4400	FUEL GAUGE ERROR AT START +750	3700	3100	2600	2500	1800	1800	1800	1800	—	2000
TANKS P.I.	3000	3000		2500	2425	2400	2400	2300	1650	1450	1250	—	1100
TANKS C.R.	—	—		—	—	—	—	—	—	—	—	—	—
TANKS S.I.	2750	2750		2450	2460	2450	2450	2300	1800	1450	1250	—	1100
TANKS S.O.	4350	4200		3700	3100	2650	2550	1800	1800	1800	1800	—	2000
TOTAL FUEL	14700	14350		12350	11075	10100	9900	8200	6750	6500	6100	—	6200
INTERVAL FUEL USED		350	2000		1275	975	200	1700	1450	250	400	—	—
EMPTY TANK WEIGHT	28115	28115		28115	28115	28115	28115	28115	28115	28115	28115	28115	28115
GAUGE WEIGHT	42815	42465		40465	39190	38215	38016	36315	34865	34616	34215	—	34315
FLIGHT PLAN RESERVE		FLIGHT PLAN FUEL		—	—	—	10385	8785	7635	—	—	—	—
6.800		DIFF. FROM FLIGHT PLAN		—	—	—	−358	−278	−328	—	—	—	—
OFF CHKS. ON TOTAL		PRESS. ALT.		14,000	25,500	34,250	36,500	36,500	36,700	26,500	17,000	4,500	
0932 1156 2:24		I.A.S. KTS.		230	208	188	182	218	214	222	220	220	
0942 1152 T/O LAND		AMB. TEMP. °C		zero (G)	−25 (G)	−30 (G)	−26 (G)	−47	−45	−18 (G)	−10 (G)	+17 (G)	

FIGURE 20. Engineer's fuel record.

compromise solution with the cabin at some lower height than is current practice would go a long way towards meeting the requirement.

11. Special Instrumentation

New types of operation inevitably raise new requirements in instrumentation, and some reference will now be made to this subject. It is logical to start with the Machmeter, the most important of the new instruments. In early Comet operations a standard service instrument was used, as shown in Fig. 18 (*a*). This was adequate as a safety indicator, but provided neither the accuracy nor the scale range needed for use as a quantitative indicator in its own right. Accordingly a modified type was introduced, as shown in Fig. 18(*b*), and a calibration accuracy of $\pm 0{\cdot}005$ Mach was achieved over the cruising range on the scale, enabling the instrument to be used for the purposes already described, thereby vastly increasing its value to the operation.

Probably the altimeter is the next most important instrument in terms of air safety. As with the Machmeter, a standard service instrument was used in the early Comet operations. It was soon found that serious mistakes in interpretation could occur when the instrument had to be read under stress, mistakes of 10,000 ft. not being unusual. In consultation with the instrument manufacturers, B.O.A.C. then sponsored a revised presentation, and this, together with a standard service pattern, is shown in Fig. 19. It will be seen that on the new presentation (Fig. 19(*b*)) the pointer indicating tens of thousands of feet has been brought outside the other ones, and that reference to it alone is sufficient for an approximate estimate of height, such as is needed during a rapid descent, for example. The two instruments shown in Fig. 19 are both indicating the same height, and it is unnecessary to stress further the improvement in clarity achieved with the new presentation, which is, in itself, a major contribution to flight safety.

Perhaps the next most important instruments are those concerned with fuel measurement, namely tank contents gauges and flowmeters. No really satisfactory method of indicating fuel contents has yet been evolved to the necessary order of accuracy, and this constitutes an urgent operational requirement. As flowmeter accuracy has been found in the light of practical experience to exceed by far that of the available contents gauges, it has become current B.O.A.C. practice to integrate the fuel flows, and to keep a continuing check between remaining fuel, and recorded flows, by this means. Fig. 20 is a typical Flight Engineer's Fuel Record sheet from the aircraft Technical Log. It will be realised that great simplification will be possible in the operating drills, given adequate instrumentation, and that this must also improve flight safety.

Finally, the importance of three standard instruments to the new operation must be stressed. These are the engine tachometers, the outside air temperature gauge and the clocks. All have been important instruments

in the past, but have now sprung into new prominence to meet the changed requirements of jet aircraft. Engine performance has become sensitive to r.p.m. to an extent previously unknown, and ambient temperature is now an important performance parameter. But time itself is the final yardstick in flight, the secret both of success and safety, and ultimately the most precious commodity in civil jet operations.

REFERENCES

1. HARDINGHAM, R. E. (1952). Maintaining Airworthiness in Operation: Eighth British Commonwealth and Empire Lecture. *Journal of the Royal Aeronautical Society*, Nov. 1952.
2. Meteorology and the Operation of Jet Aircraft: Discussion Meeting reported in the *Quarterly Journal of the Royal Meteorological Society*, Vol. LXXVIII, No. 337, July 1952, particularly the contribution by Capt. A. M. A. MAJENDIE.

Discussion

I. R. LIDDELL (B.E.A.): It was perhaps not inappropriate that the lecture should be given at Glasgow, for he thought it likely that the first route in Great Britain on which there would be jet operations might well be between Glasgow and London, and before very long there would at least be propeller-turbine operations.

There was a growing tendency for short-range operations to be made with larger and larger aircraft in order that more and more passengers might be carried economically and the tendency would be to adapt the large, long-range jet aircraft of the future to short-range operation. Could they have Captain Majendie's views on the operating technique which might have to be adopted in such cases?

F. B. GREATREX (Rolls-Royce Ltd., *Associate Fellow*): The choice of cruising height had been mentioned, but he wondered if, under turbulent conditions, Captain Majendie would fly at a slightly lower altitude, particularly as the normal cruising height was near to the absolute ceiling? He would like to know a little more about how he actually selected that cruising height. Various methods had been commented on. For example, if they were going to fly at a particular r.p.m. and speed they could choose an altitude at which, if they reduced speed to the best climbing speed, they had a certain potential rate of climb. Another matter which had to be taken into account was the g available for manoeuvring. Even if they were prepared to lose height temporarily, they had to be able to manoeuvre a little. The figure which he had heard mentioned for potential rate of climb was 250 ft./min. He would be interested in Captain Majendie's views on that figure.

Another point mentioned by the author was that most of his operations so far had been in the troposphere; also, that if he had more cabin differential available he would have more freedom of operation. Was there any physiological limit to the height at which he would like to fly? There had been a mention of 42,000 ft. as a figure at which if anything did happen the passengers would not survive at all.

Then there was the question of instrumentation. What sort of a flowmeter would Captain Majendie really like? Was it the sort of thing that a fighter pilot would like, to tell him if he went on flying at the r.p.m., altitude, etc., he was using, how much longer time he had left to fly at those conditions with the fuel remaining in his tanks? That was quite a complicated sum for a flowmeter to do. Was that the sort of thing he was interested in?

A. W. BABISTER (University of Glasgow, *Associate Fellow*): The lecturer had talked about take-off and had presented various diagrams of take-off distances and take-off run and had also shown how jet aircraft compared with other types, but he had not said anything about take-off in a cross wind. With an aircraft such as the Comet there would be only a very small thrust moment to counteract the yawing moment due to a cross wind. How did they compare with some American civil aircraft with the engines very much outboard?

J. R. NUTTER: There had been two take-off accidents with the Comet; would Captain Majendie like to comment on that? Were they in any way attributable to lack of " feel " associated with power-operated controls, or did they arise from a major change in technique with the jet aircraft?

L. POLLI: Could Captain Majendie explain the basic difference between operating a turbo-jet of the type in the Comet and propeller-turbine engines?

CAPTAIN W. ST. CLAIR REID (B.E.A.): Most of the equipment had been covered, could Captain Majendie tell them anything about de-icing?

J. M. MARSHALL: What was the comparable take-off length of run for the jet aircraft compared with a piston-engined aircraft of the same all-up weight?

N. E. ROWE (Blackburn & General Aircraft Ltd., *Fellow*), *contributed*: The lecturer had done a great service in showing how civil jet operations had led to a completely new approach to so many of the techniques of operation which had been in existence for many years and had been thoroughly accepted.

An outstanding example was the technique of flying at constant lift coefficient, with the associated need to determine the weight of the aircraft accurately at all stages in flight and also, to adopt new means for measuring performance in flight. One of the most interesting developments was the adaptation of the Machmeter as an operational instrument and its use, together with observed total temperature, to find the true air speed and the true ambient temperature.

The lecturer made a very important point on the need to improve terminal forecasting facilities. He said that he thought it would be a better investment to spend

money on this at this time rather than on the effort to build an adequate network of upper air meteorological reporting stations. That was a new standard, since it was vital that for most economical and safe use the jet-driven aircraft should not descend unless it was definite that alighting could be made. It would be interesting to hear whether any steps had been taken to provide this sort of information and what difficulties were foreseen?

In his comments on air traffic control, the lecturer referred to the advantage of ability to lose height quickly, and hence the need for good air brakes on jet air liners. He would like to couple with that the lecturer's comments on the desirability of higher cabin pressures, since he presumed that with the present level of cabin pressurisation, rates of descent must be restricted from those possible with the best air brakes. Would Captain Majendie give his opinion on the maximum rate of descent he foresaw in the future, since that set a standard of pressure differential between cabin and ambient conditions at the maximum cruising height? Would the operator be prepared to forego windows for passengers in the aircraft in order to minimise the structural penalty of maintaining surface pressure inside the cabin at all heights?

The lecturer made very interesting comment on the route application of the civil jet air liner, which he said could only justify itself on long distance trunk route operations. That was an extremely important matter since it implied that for the shorter stages the propeller-turbine would be pre-eminent. However, that question must be influenced to a degree by the standard of fuel consumption attained, and if there were marked reduction in the fuel consumption of the jet engine, the lecturer's opinion might be modified.

He gathered from the remarks about fuel contents gauges, that the operator was now satisfied with the integration of fuel flows measured on the flowmeter, and hence the need to improve fuel contents gauges was not so great. Perhaps he would confirm this?

D. W. G. KEESING (Senior Pilot, Tasman Empire Airways Ltd.; New Zealand Division of the Society), *contributed*: It would be very interesting to see the "performance envelope" for the Comet as this graph was not mentioned in the paper.

Under the heading "climb and descent" mention was made of climbing speeds relevant to turbulence. A fuller explanation of this would be of interest.

Some information on icing problems with reference to engine, airframe and fuel would also be welcome.

It would be of interest to know if there were any question of gyroscopic effect of the turbines affecting the manoeuvring of the aircraft, and whether it imposed any severe structural strains under conditions of turbulence.

W. H. DUNN (Chief Performance and Power Plants Engineer, Civil Aviation Branch, N.Z. Air Department; *Associated Fellow*, New Zealand Division), *contributed*: The paper was most interesting and one which had been much wanted as so little information was available on this type of operation. There were, however, one or two points which he thought would be of help in assisting anyone to make an assessment of a proposed operation of the Comet on any particular route.

Could some data be included in the report giving the fuel used on the climb and the distance covered for various take-off weights? Also, an explanation of what happened when using the cruise tables shown in Figs. 9 (*a*) and (*b*) when the take-off weight was restricted and, therefore, less than that used in preparing the tables. Were there similar tables for various aircraft weights at the start of the cruise, or did the engineer use the table shown in Fig. 9(*a*), making adjustments to the time and scale for the different aircraft weights? What were the contingencies normally allowed for in assessing the fuel requirements for a flight with, and without, the availability of an alternate aerodrome? What was normally the latest stage in the flight when a decision had to be made if the alternate were to be used?

Captain Majendie's Reply

MR. LIDDELL: Current experience was quite simple. If they were operating a jet aircraft on short runs the most economic thing to do was to climb as high as possible and then descend, even though they had not reached normal cruising level. He had done it, for example, between London and Prestwick. In that case he had climbed to Manchester and then descended from there to Prestwick. It was much better to do this, as level flight below the optimum cruising range was definitely uneconomic.

MR. GREATREX: There seemed to be some confusion in the speaker's mind about what he had said on turbulence coupled with the selection of cruising height. The operation as they carried it out today was not to select a cruising height as such, but to allow the aeroplane to choose its own. They were flying to an indicated air speed and set engine r.p.m., and if they were flying in stable air under such conditions the aeroplane would take up its own level. If, under cruising conditions, they encountered turbulence and the speed was above that for operation in turbulence, then they would change to a slower cruise. They had actually two standard cruises, one at 30 per cent. above and one at 15 per cent. above the speed for minimum drag. If they were flying on the 1·3 cruise and encountered turbulence, they dropped back to 1·15 and allowed the aeroplane to drift up to the new altitude. It would be difficult to make any attempt to select the operating altitude under those conditions; if anything they would tend to operate a bit higher in turbulence, not lower.

As far as manoeuvre was concerned, as long as the aircraft control characteristics were satisfactory there was no reason why they should not operate around the minimum drag conditions. Potentially the rate of climb did not enter into it because they did not mind what

height they were at. If they were operating at minimum drag conditions, and turbulence arose to cause the aircraft to drop below minimum drag they then had insufficient thrust to meet this increase, and the aircraft would descend a bit: more thrust then came from the engine and conditions were re-established. Potentially the rate of climb did not matter, as it was not like a level flight cruise with a piston-engined aircraft.

On the physiological limits, he thought that the figure of 42,000 ft. was a slightly unreal concept. The aeroplane could be an extremely dangerous thing "if something should happen," to quote Mr. Greatrex's words, if it were only at 500 ft. The wings could come off, or the tail fall away, or it could fly into a mountain. Catastrophies could happen, but there were certain things they could not tolerate in the air, and now that they were flying at the heights that they were, a catastrophic failure of the pressurisation was one of the things they had to have complete protection against.

Of flowmeters, he would say that what the pilot really wanted was an accurate indication of actual engine consumption at any moment, together with an equally accurate indication of the fuel remaining in the tanks. Integrating flowmeters were being experimented with, as the difficulties of providing adequate tank contents gauges had not yet been overcome. The type of device suggested could be extremely misleading. If one were operating to a reduced fuel margin, then such a device could show considerably less time to go than the estimated time needed to reach destination if read shortly before starting the descent. One would know it was going to take another 50 minutes and the indicator would show that one could only keep going for another 40 minutes, but one might still be able to arrive at the destination with a half-hour's fuel reserve. What was wanted was an accurate indication of the fuel left in the tanks and to do the calculating for oneself. At present there was no satisfactory fuel contents gauge covering the very large quantities of fuel they had to handle, to an acceptable standard of accuracy under the extreme range of operating conditions encountered.

MR. BABISTER: On the other side of the Atlantic they had seen a tendency to thin highly swept and rather elastic wings, and clearly an engine mounted outboard in a case like that demanded tremendous rudder control to hold it. Design in Great Britain had been to group the engines close in to the centre line of the aircraft, and relatively modest tail surfaces were needed to control the engine drag. It was possible on the Comet to cut out an engine and still complete a take-off without the application of rudder. It was not usual practice to use asymmetric engine thrust on a large aircraft with a steerable nose wheel to counteract cross-wind effects on take-off. If the tail fin and rudder could be kept small, as was possible with jet-engines installed close to the fuselage, cross-wind effects could be kept down, as they had been on the Comet. He felt that the American tendency to position engines a long way out on the wing was to be deplored from the point of view of the pilot.

MR. NUTTER: He rather suspected someone would ask about the Comet take-off in relation to the accidents. They were not due to any abnormal peculiarity of the type; they were satisfied of that. Something similar had been encountered before. One of the versions of the war-time Liberator was peculiarly susceptible to tail-down in take-off, and was for some time fitted with a projection at the back which turned on a light on the instrument panel if the nose were too high. The current flying technique on the Comet was to keep the nose wheel in contact with the ground until reaching the unstick speed. That did seem to be the answer. It was probably true to say that the cause of those accidents was related to lack of feel in the power-operated flying controls, but the provision of such a simple solution augured well for the future. He saw no reason to fear that that type of accident would occur again.

MR. POLLI: He had deliberately and carefully confined himself to those things of which he had had experience. He had no experience of propeller-turbine operation, so he would not attempt to answer this question.

CAPTAIN REID: The de-icing on a Comet was extremely simple. It consisted of a feed from the main engine compressors supplying hot air. It was taken from a ring round the combustion chambers. This hot air was then fed down ducting to the area around the engine intakes, to the leading edges of the wings and the leading edges of the tail surfaces, finally spilling over the control surfaces. All important parts were thereby provided with thermal protection against icing, the only exception being the pilots' windscreen, which had a conventional fluid system.

MR. MARSHALL: There was no simple answer to Mr. Marshall's question about comparable take-off runs. It depended on weight, temperature, wind, altitude and many other conditions. On their current operation they had to meet the same airworthiness margins as would the piston-engined machine. He could not give any value, the variations would be tremendous. It had perhaps been unfortunate that the introduction of the Comet into service had coincided with the introduction of the new British Regulations on take-off performance, and to which it had been certificated. Interested third parties had undoubtedly become confused on a number of occasions between the characteristics of a jet aircraft, and the requirements of the new Regulations.

MR. ROWE: No attempt had yet been made to carry out a regular operation on the basis that the aircraft would be committed to land once descent had been started. First steps in this direction had, however, been taken in one of two selected instances, where the regular alternate might be denied to the Comet once it had passed a certain point on its descent, but in each of these cases a nearby emergency alternate was available in the event of a wave-off diversion. It was hoped to extend this principle as further experience was gained.

The importance of being able to lose height quickly could not be over-estimated and provided a valuable

simplification to the air traffic control problem for approach at busy airports. It was not easy to quote a maximum figure for this requirement, but he felt that future designs should aim at a figure of about 5,000 feet per minute. In order to use such a figure the pressure differential would have to go up a little bit more than at present, but this would also bring other advantages in its train, such as increased passenger comfort, and so on. Clearly it would be much easier to achieve this without too large a weight penalty if windows did not have to be provided in the structure: unfortunately he very much doubted whether any passengers would wish to travel in a windowless air liner, and the absence of windows, apart from denying the sight of magnificent cloudscapes, might also induce claustrophobia in the more nervous passengers.

On the subject of flowmeters, and fuel measurement in general, it must be pointed out that the integration of fuel flow was a most unsatisfactory method of arriving at fuel remaining. Not only did errors, when expressed in terms of the fuel remaining, increase as a flight progressed, but also this method provided no certain information upon the disposition of remaining fuel in the various tanks. It was becoming increasingly important to control fuel usage with swept-wing aircraft, both to control the centre of gravity, and to provide stress-relief on the wing structure. Furthermore the integration of fuel flows was valueless in determining defects in the fuel system, such as leaking tanks, or the unintended flow of fuel from one tank to another. For these, and other reasons, it was true to say that the development of accurate and reliable fuel tank contents gauges was one of the most urgent and vital requirements in aviation to-day.

MR. KEESING: The main purpose in the paper had been to illustrate the general principles and practical techniques of this type of operation. Such performance data as had been used was for illustrative purposes only, and it would clearly have been impossible to provide a comprehensive account of Comet I performance as such.

The reference to climbing speeds and turbulence was to draw attention to the fact that the best performance air speed on the climb might be unacceptably fast for use in bumpy conditions, and that provision must be made for a slower air speed technique for use in those circumstances.

So far as icing was concerned no unusual difficulties had been encountered either with the engines or the airframe. At one stage some difficulties were experienced with the fuel system, particularly at the low pressure filters. Those difficulties had been successfully overcome on the Comet I, but undoubtedly there were going to be major problems to be solved in this connection on future jet transports, which would fly for longer periods in colder air than was usual at present.

No gyroscopic effects were noticeable in flight due to the turbines, and no abnormal structural problems had been encountered for this reason.

MR. DUNN: Performance figures quoted in the paper should not be used in assessing possible operations with jet transports, as newer aircraft showed considerable improvements in performance on many of the figures quoted, which were strictly related to Comet I operations.

The tables shown in Figs. 9(*a*) and 9(*b*) were used in practice for operations at all weights, and had been designed for that purpose. As vertical columns in Fig. 9(*a*) represented increments of one hour, and horizontal lines increments of two minutes, it was a simple matter to enter at any weight, and determine the weight for any subsequent interval of time.

It was too early yet to attempt any statement in general terms on the subject of jet fuel reserves. The principle adopted in B.O.A.C. had so far always been to carry enough fuel to enable a safe diversion to be made after a missed approach at the intended destination.

A DISCUSSION ON FATIGUE

A Full-Day Discussion on various aspects of Fatigue was held by the Royal Aeronautical Society on 27th March 1953 at University College, Gower Street, London, W.C.1. Four papers were given during the day: "Fatigue" by H. L. Cox, M.A., F.R.Ae.S., and "Fatigue from the Metallurgist's Point of View" by E. R. Gadd, F.I.M., during the Morning Session; and "Fatigue in Engine Design" by F. M. Owner, C.B.E., M.Sc., F.R.Ae.S., and "Fatigue in Aircraft Design" by B. E. Stephenson, A.F.R.Ae.S., during the Afternoon Session.

Major P. L. Teed, F.R.Ae.S., Deputy Chief of Aeronautical Research and Development, Vickers-Armstrongs Ltd., presided during the Morning Session, and Dr. E. S. Moult, F.R.Ae.S., Chief Engineer, de Havilland Engine Co., took the Chair for the first half of the afternoon, and Mr. B. S. Shenstone, F.R.Ae.S., Chief Engineer, British European Airways, for the second half.

Each paper was followed by a Discussion and there was a general discussion at the end of the day. Mr. W. Tye, O.B.E., F.R.Ae.S., acted as rapporteur for the Discussions.

Fatigue

by

H. L. COX, M.A., F.R.Ae.S.
(*National Physical Laboratory*)

IN OPENING THIS DISCUSSION on fatigue it seems that my first duty should be to state what fatigue is—I wish I knew. Just as Alice knew that if you drank much from a bottle marked "Poison" it was pretty sure to disagree with you sooner or later, so do we know that if a component be subjected too often to too high a load it is pretty certain to develop fatigue cracks sooner or later.

It is important to remember that Lewis Carroll was by profession a mathematician, because in a mathematical sense even these heavily conditioned statements should in fact convey a great deal. "Too often to too high a load" infers range of loading, and furthermore it implies a connection between range of load S and cycles N, by which in general N may be expected to increase as S decreases. "Sooner or later" indicates that N is not a unique function of S, and "pretty certain" provides for those pieces which, like certain batteries, obstinately go on long after the others have stopped. Then, lastly "to develop fatigue cracks" is a careful avoidance of a direct reference to failure.

We may put a notch in a testpiece, we may make the notch as sharp as we can; we may even start a crack by fatigue or otherwise: we still have to consider whether under the loading applied the notch will crack, or the crack propagate. If we ask the physicist he will point out that the question is one of balance between volume and surface energy and refer us to Griffith's classic paper. If we ask the mathematician he will observe that a notch, or even a crack, is after all only a stress-free boundary of special form; that the plainest of plain testpieces still has a stress-free boundary; and that therefore the question in its most general form is how modification of the form of any boundary may reduce the total energy. Then, since the boundary is never completely free from chemical contamination, we should need to consult the chemist and to consider in what degree all fatigue should be regarded as corrosion-fatigue.

Corrosion-fatigue is a very messy subject, and having warned you all not to forget it, I feel entitled now not to soil my talk with it. On the other hand ordinary fatigue cracks are much cleaner and sometimes rather pretty, so I shall talk a little more about crack propagation. Nor shall I apologise for talking about the development of cracks before considering how they may start; after all, if the crack does not propagate, why should we worry about it at all?

Figure 1 shows a crack formed at the root of a V-notch 0·2 inch deep with a root radius of two thou. after 50 thousand reversals of alternating direct stress. It was about two thou. deep and very narrow. Two other exactly similar pieces were tested under the same load range and sectioned after they had endured 25 million and 100 million reversals respectively. Each was similarly cracked, the former to a depth of one thou. and the latter to three thou.

These results, obtained by Mechanical Engineering Research Laboratories and the National Physical Laboratory, reinforce user experience of the type first documented by Bacon, but it is also a common observation in laboratory testing that some cracks simply refuse to spread. This refusal is certainly very puzzling because the elastic stress concentration at the root of these fine cracks is terrific; although metallurgists at the Royal Aircraft Establishment and in the United States have shown that the material ahead of the crack does deform plastically, it is difficult to believe that this

FIGURE 1 (*left*). Crack at root of sharp V-notch.

FIGURE 2.

plastic deformation can continue throughout hundreds of millions of reversals, preventing the high stress concentration and yet not damaging the material.

If at present we cannot answer the question when a crack should spread, can we attempt the questions how and why? Is the determining influence the range of direct stress tending to pull the crack open or is it the shear stress tending to slide one part over the other? Does the crack propagate along planes subjected to maximum tension or maximum shear? Sometimes the answer is quite definite; Fig. 2 is an example from service of a fatigue crack in a helical spring stressed in pure shear. The crack started at a slight surface flaw and developed on the surface of maximum direct stress, following it very closely indeed. But suddenly the mode of propagation alters, and there is a strong indication that the propagation is now on planes of high shear. This behaviour is typical of aluminium alloys: in tests on D.T.D.646 sheet 0·08 in. thick, Phillips and Fenner had many excellent examples of a tensile crack starting from a slight stress concentration, changing to double shear and then changing again to single shear. The last stage may have been propagation under the mean static load, although it seemed to me that the pulsation of load was still having a vital influence. There is not time to go into that further, although in detail the cracks had other marked characteristics, which were most intriguing.

In an attempt to decide which mode was preferred, Field and I made some few tests on square pieces of mild steel under combinations of bending and torsion. Under pure torsion the maximum stress is along the centre of the face and Fig. 3 shows the crack following the shear plane under a loading which was nearly pure torsion. Under pure bending about one diagonal of the square the maximum stress is at the edge and Fig. 4 shows under almost pure bending the crack starting at

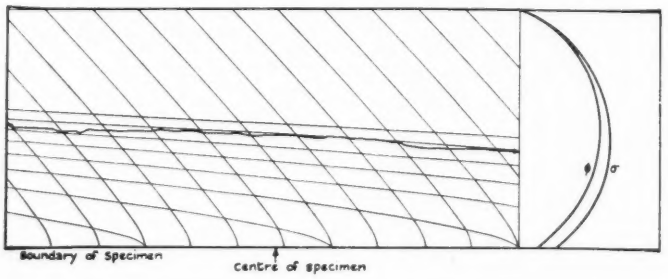


FIGURE 3. Distributions and directions of maximum direct stress and maximum shear stress and course of fatigue crack.

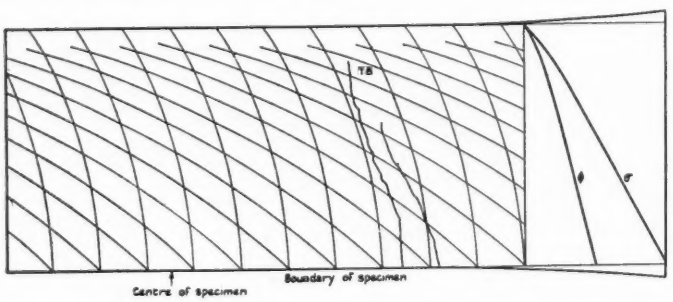


FIGURE 4. Distributions and directions of maximum direct stress and maximum shear stress and courses of fatigue cracks (showing crack on each of two faces of one specimen).

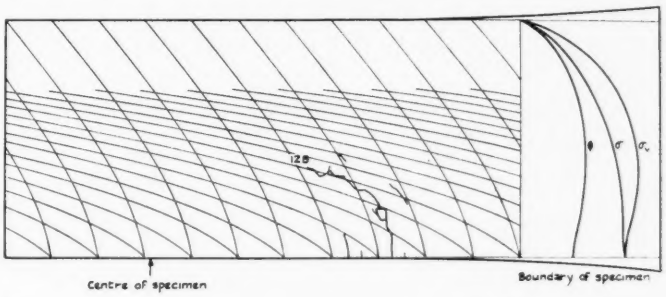


FIGURE 5. Distributions and directions of maximum direct stress and maximum shear stress and courses of fatigue cracks (showing cracks on one face of one specimen).

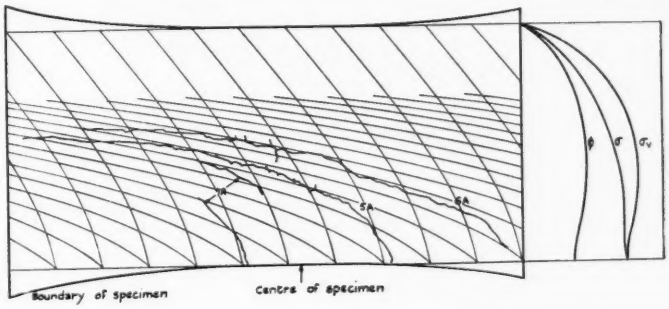


FIGURE 6. Distributions and directions of maximum direct stress and maximum shear stress and courses of fatigue cracks (showing cracks on one face of each of three specimens).

the edge and following the plane of maximum direct stress. In Fig. 5 under a nicely balanced system we again have tension cracks at the edge—and incidentally several of them—while the one which propagates farthest seems to wander off into the shear direction. Yet in Fig. 6 in another testpiece (6A) under the same loading there are cracks starting and propagating on the shear plane; in a third piece (5A), a crack mainly shear but either running to the edge or possibly even starting there and tending there to the tensile mode; while in this fourth (7A) there is very little doubt that there are both types of crack formed simultaneously. That again is typical of fatigue—I know no other subject where one may do so many tests and ascertain so very little.

While on this question of mode of cracking I cannot resist showing Fig. 7. This was a piece, pierced by a diametral hole, tested under a high range of pulsating compression. The plastic deformation in shear under the high stresses at the sides of the hole resulted in the originally round hole closing up almost completely; but the main crack occurred by splitting above and below the hole due to the range of pulsating tension there. However all I feel prepared to conclude at present is that the fatigue crack is likely to start in a region of maximum stress range, that it may develop in tension or in shear with a slight preference for the former mode.

Provided that the survey is confined within fairly strict geometrical limits this qualitative statement may be extended quite usefully in a quantitative way almost regardless of metallurgical differences. For instance, Fig. 8 shows the results of tests under combined bending and torsion of testpieces pierced by diametral holes. In such a test the maximum stress or stress range must always be set up somewhere in the periphery of the hole and this stress is a principal stress with the other two zero. Therefore failure occurs always under uniaxial stress and the behaviour ought to be determinable from geometrical considerations alone. These seven materials all conform as closely as one may expect, and more recently tests on three or four other steels have yielded results conforming still more closely. Similar data for aluminium alloys are now being compiled.

It may be asked what in fact Fig. 8 does represent. The theoretical curve represents constant maximum stress in the periphery of the hole for all combinations of bending and torsion. The experimental points represent for each stressing condition the highest loading at which one or more of eight or more testpieces endured at least 10 million cycles without failure; that, and no

FIGURE 7. Test piece split longitudinally under repeated compression.

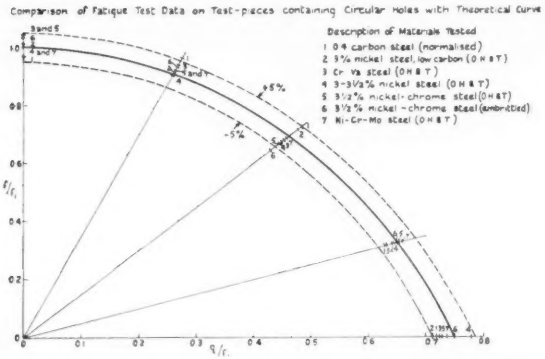

Comparison of Fatigue Test Data on Test-pieces containing Circular Holes with Theoretical Curve
Description of Materials Tested
1 0·4 carbon steel (normalised)
2 3½% nickel steel, low carbon (O H & T)
3 Cr Va steel (O H & T)
4 3-3½% nickel steel (O H & T)
5 3½% nickel-chrome steel (O H & T)
6 3½% nickel-chrome steel (embrittled)
7 Ni-Cr-Mo steel (O H & T)
+5%
-5%
f/f1
q/f1

FIGURE 8. Effect of holes on strength of materials under complex stress systems.

more than that. Any extension of that statement must be conjectural.

The stresses computed by elastic theory at which these pieces failed were *all* about 15 per cent. greater than the stresses needed to fracture plain pieces, or (the other way round) the practical stress concentration proved to be about 87 per cent. of the theoretical. Since the tensile strengths of the materials ranged from 35 to 80 tons/in.^{2}, this uniformity of behaviour may seem a little surprising; all the materials proved very nearly fully notch sensitive to the hole about 0·05 in. in diameter in a piece $\frac{1}{2}$ in. in diameter.

If the region subjected to stress concentration is big enough it seems inevitable that the piece should prove fully notch sensitive; in other words I relate the reduction of notch sensitivity below unity with the stress gradient at the point of stress concentration. This hypothesis, originally propounded by Neuber, involves the conception of fatigue occurring not by failure at a point but in a region which may be typified by a characteristic length in the material itself. Further exploration of this conception in relation to microstructure is urgently needed; all that can be claimed with confidence at present is that the size of the region is not determined by grain size, although it may be affected by it.

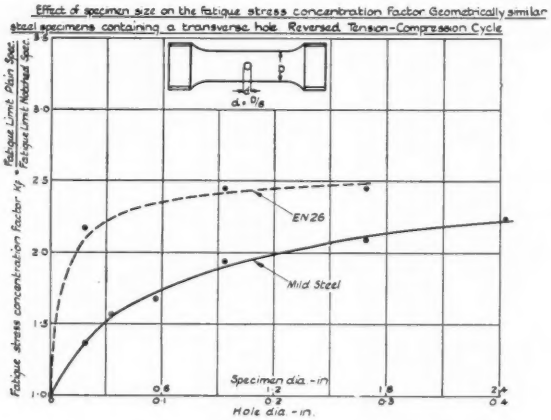

Effect of specimen size on the fatigue stress concentration Factor Geometrically similar steel specimens containing a transverse hole. Reversed Tension-Compression Cycle
Fatigue Limit Plain Spec. / Fatigue Limit Notched Spec.
Fatigue stress concentration Factor kf
EN26
Mild Steel
Specimen dia. - in
Hole dia. - in.

FIGURE 9. Summary of results of size effect. Investigation of steels.

Figure 9 shows how the stress gradient effect results in a variation of behaviour with size. The upper (dotted) curve is for a 60 ton steel, and at $\frac{1}{2}$ in. diameter the notch sensitivity is again over 80 per cent. (The comparison with Fig. 8 is not quite direct, because these tests were made under alternating direct stress, whereas the others were made under alternating bending.) For mild steel (the lower full line curve) the notch sensitivity does not reach 80 per cent. until the size reaches 2·4 in. with a hole 0·4 in. in diameter. However for mild steel and indeed plain carbon steels in general, the concept of stress gradient does not suffice, because in these materials the stress-strain relationship even for a plain testpiece uniformly stressed is non-linear and continues so throughout many millions of cycles. Attempts are being made to assess the effect of this hysteresis, and it has already been shown by Tapsell and Forrest that when allowance is made for the curvature of the stress-strain relationship the fatigue of mild steel under alternating bending is fully consistent with its behaviour in fatigue under uniform direct stresses. In this diagram therefore it is conceivable that similar reassessment of stress would raise the curve for mild steel to that for the 60 ton steel.

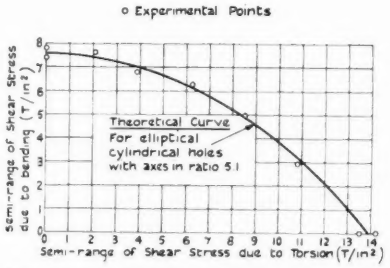

o Experimental Points
Semi-range of Shear Stress due to Bending (T/in²)
Theoretical Curve For elliptical cylindrical holes with axes in ratio 5:1
Semi-range of Shear Stress due to Torsion (T/in²)

FIGURE 10. Results of fatigue tests under combined alternating bending and torsion on " Silal " Cast Fe.

At the left of the figure the curves have been drawn to notch sensitivity zero at zero size. Of course for this we have no experimental evidence and the best that can be done is to attempt to approach the limit a little more closely. The few results that have been obtained both on steels and aluminium alloys do suggest that a very small hole reduces the strength only very slightly. A most useful body of evidence on very small stress raisers is afforded by the results of tests on cast-iron. Fig. 10, for instance, shows the results of tests under combined bending and torsion on one typical sample. On the assumption that failure always originates at the graphite inclusions, and following the same train of reasoning as for drilled holes, it is possible to estimate the average effective shape of the graphite inclusions and hence to deduce the theoretical stress concentration factor (s.c.f.). Agreement with theory is very good and moreover the mode of fracture of these pieces was also in good accord with theory. The deduced s.c.f. for direct stress or bending is about 11 or 6 for shear, corresponding to a nominal failing stress at the ends of the graphite flakes of ± 82 tons/in.^{2} Probably the actual stress range on the ferritic material is considerably reduced by plastic

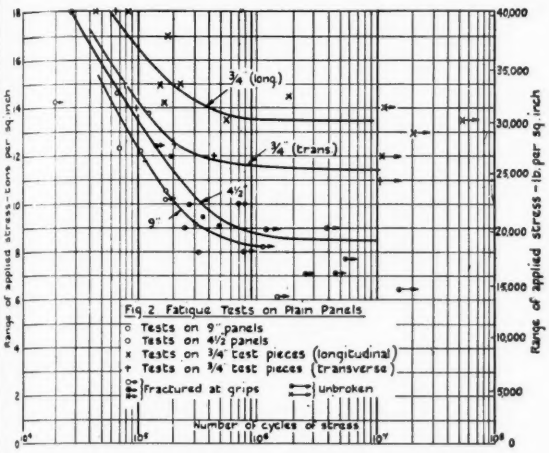

Range of applied stress - tons per sq. inch
Range of applied stress - lb. per sq. inch
3/4" (long.)
3/4" (trans.)
4½"
9"
Fig 2 Fatigue Tests on Plain Panels
Tests on 9" panels
Tests on 4½ panels
Tests on 3/4" test pieces (longitudinal)
Tests on 3/4" test pieces (transverse)
fractured at grips
unbroken
Number of cycles of stress

FIGURE 11. Fatigue tests on plain panels.

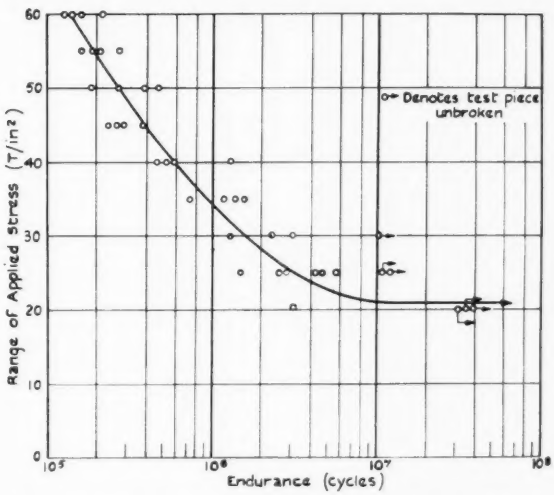


FIGURE 12. Results of repeated loading tests on helical springs of 0·128 in. diameter. Hard drawn steel wire.

deformation as for straight carbon steels, and that expectation is supported by the high internal damping exhibited by cast-iron. However it seems improbable that this will account for the whole of the increase above the order of strength to be expected for the ferritic material, and therefore we may conclude that the behaviour of cast-iron is at least qualitatively consistent with the concept that the fatigue strength is considerably increased when the high stress is developed only in a very small region.

All that has been said so far is based on the concept of endurance limit, the highest range of stress at which one or more of a number of similar pieces will endure a specified number of cycles of stress without actually failing. What evidence is there that even this strictly limited concept has a real meaning?

Figure 11 shows the results of tests on a number of hard aluminium alloy plates, all 14 s.w.g. and of various widths. It will be agreed that it would be difficult to state with confidence a precise endurance limit at any ordinate for any of the four series of tests, and yet regarding the results as a whole it does appear safe to conclude that the material is slightly stronger one way than the other and that its strength decreases as the width of the test-piece is increased, tending possibly to a limit. These results are shown deliberately, in order to emphasise the difficulties inherent in fatigue testing; there are upwards of 50 tests recorded, made in two or possibly three

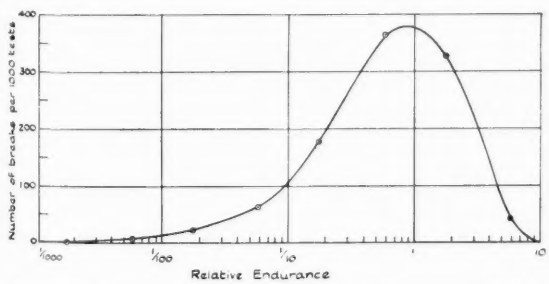


FIGURE 13. Scatter of endurance for test pieces equally likely to fail in each and every cycle.

different machines. It requires very considerable care to ensure that such results should be strictly comparable, and it is most unwise to compare endurances unless a number of repeat tests is made. In Fig. 12, for instance, are the results of 40 tests on helical springs tested in groups of four. The tolerances on such springs made in one batch from hard drawn steel wire are at least as close as those of ordinary machined testpieces and the loading conditions—the tests were made all in one machine—are better than average; yet the scatter of endurance is quite wide, particularly when the average endurance is some millions.

In dealing with fatigue it is always essential to regard the results of tests or of experience in practice in a statistical light. Fatigue could be—I do *not* say that it is—a purely statistical effect and still we should expect a fairly well defined $S-N$ relation. Fig 13, for instance, shows the scatter of endurances to be expected if in any cycle from the first to the umpteenth the chance of failure is $1/N$. The average endurance would be N and 70 per cent. of the results would lie within the range $N/3$ and $3N$. If the chance of failure—again assumed to be the same in each cycle—were $(1/n)^{f(s)}$, where $f(s)$ is some function of the stress range S, the average endurance at S would be $N=n^{f(s)}$, or $\log N=f(s)\log n$. Then a straight relationship between S and $\log N$ would imply $f(s)=S_m-S$ or a log-log relation $f(s)=\log(S/S_m)$.

The range of S which can be covered in a series of fatigue tests is never wide enough to afford—in relation to the inevitable scatter of results—a worthwhile comparison between the linearity of the $S-\log N$ plot and that of the $\log S-\log N$. The latter is probably preferable but it is interesting to note that by the former method the intercept on the axis of no stress often lies between 10^9 and 10^{15} cycles, tending to be lower for pieces with stress concentrations than for plain pieces. The figures already shown for plates of hard aluminium alloy and for helical springs both indicate abnormally low values of this intercept between 10^7 and 10^8 cycles,

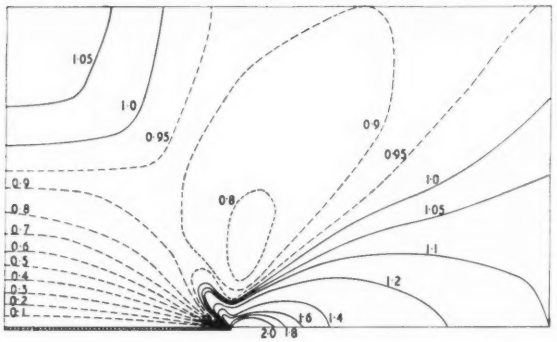


FIGURE 14. Stress distribution round an infinitely sharp crack under shear.

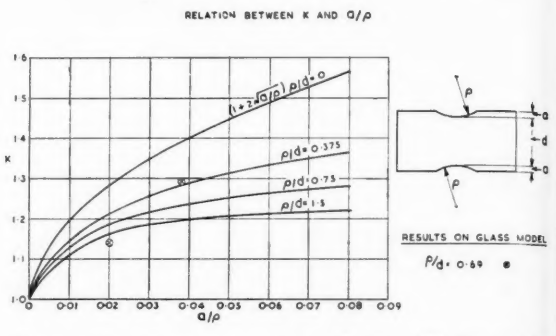


FIGURE 15. Stress concentration factors (K) for notches bounded by circular arcs—plane stress.

whereas several cast-irons tested in bending gave values nearly all round 10^{10} and several steels of normal types and aluminium alloys tested in bending often give 10^{13} to 10^{15}. I note this not because the intercept itself can have any real meaning, but as a convenient means of calling attention to the slope of the $S-N$ diagram to which ordinarily too little regard is paid. The other intercept on the $N=1$ axis which is independent of the method of plotting is also interesting, but it cannot be properly discussed until further results at very low endurances (10 to 10,000 cycles) shall have been obtained.

I want to conclude these opening remarks by referring once again to stress concentration effects. Fig. 14 shows one of the highest stress concentrations possible, the stresses round an infinitely sharp crack under shear. The infinity of stress here falls away so rapidly that even the contour 5 is scarcely distinguishable, while 2 itself is quite modest. Indeed if you look at this diagram with one eye shut you may possibly conclude that an infinitely sharp crack is an excellent thing since it reduces the stress over so large a part of the field. On the other hand very slight stress concentrations are much less mild than might be expected. In Fig. 15, for instance, are some values of stress concentration for shallow circular arcs in the sides of a testpiece; the stress concentration factor is round about 1·2 even when the depth of the groove is only 2 per cent. of the width. In contrast to the highly localised stress concentrations which result from sharp corners the stress increase due to these shallow grooves is liable to spread over a considerable region and therefore the effective notch sensitivity of the material may well approach unity. In consequence of this difference testpieces containing very small diametral holes, both in steels and in aluminium alloys, not infrequently fail at the ends of the fillets where the nominal s.c.f. may be about 1·2 to 1·3, rather than at the hole where the nominal s.c.f. is 3.

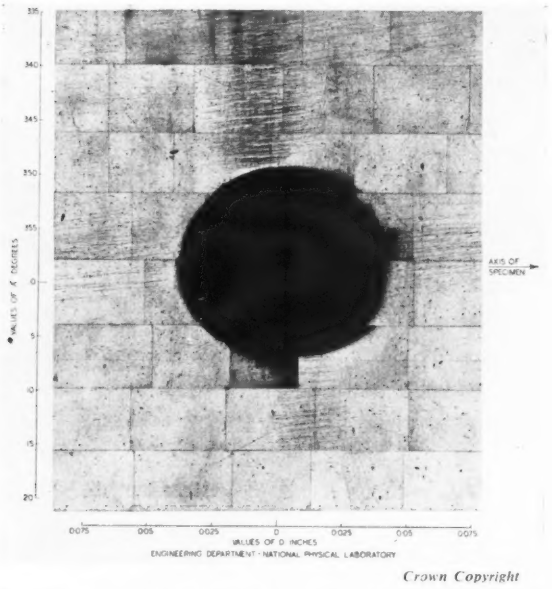


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FIGURE 16. Aluminium Crystal specimen HFO 12D2. Panorama of slip distribution round drilled diametrical hole after test 4A (first test after annealing)—51,000 reversals at nominal shear stress of 0·87 tons/in.^{2}, alternating torsion.

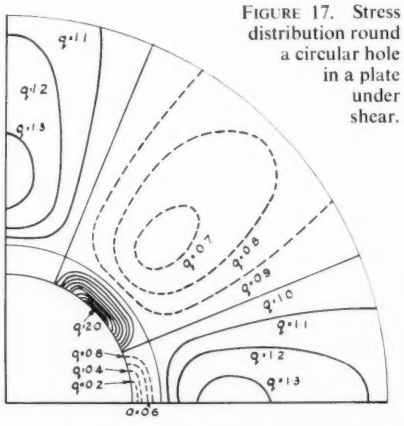

FIGURE 17. Stress distribution round a circular hole in a plate under shear.

Finally I want to show two examples of the effect of stress concentration in single crystals of aluminium. These were round pieces about 0·6 in. in diameter pierced by a diametral hole about 1/16 in. in diameter

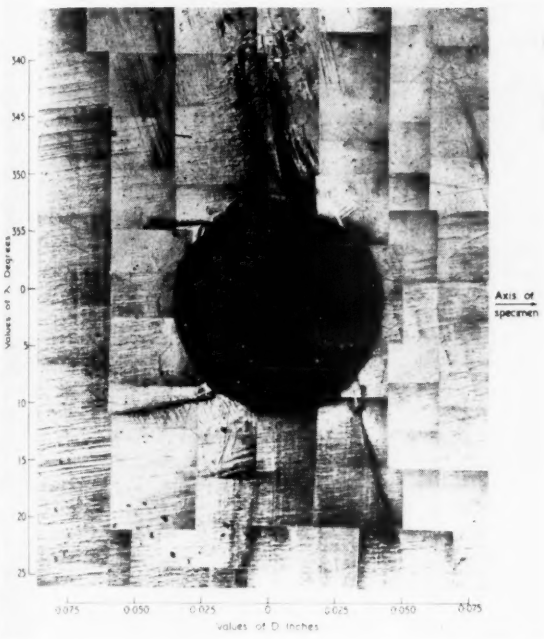

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FIGURE 18. Aluminium Crystal specimen HFO 12D2. Panorama of slip distribution round drilled diametrical hole after test 6A (third test after annealing)—100,000 reversals at nominal shear stress at $\pm 1{\cdot}19$ tons/in.^{2}, alternating torsion.

and they were tested under alternating torsion. Fig. 16 shows the effect of a short run at a stress range not much below the fatigue limit of a plain testpiece. I was puzzled by this result—there was slip where I did not expect it and no slip where I did expect it. However we were interrupted and it was some years before I got round to computing the stress distribution round the hole (Fig. 17). Then I realised that the slip bands merely showed where the *average* stress was high. Indeed moderately heavy slip under $\pm 1{\cdot}05$ tons/in.^{2} and none under $\pm 0{\cdot}70$ tons/in.^{2} is fully in accordance with our general experience of single crystals of aluminium. Fig. 18 shows the appearance of the testpiece after fracture under a stress range which would have broken a plain testpiece. The heaviest slip is still above and below and at the sides of the hole where the *average* shear stress is highest, but three of the four cracks spring from the points in the periphery of the hole where the elastic stress concentration should be greatest. We had this result in 1936 together with some other evidence which generally supports it, but (as I said) we were interrupted. Indeed this last panorama was made only recently, and in apologising for its poor quality I would remark that the polish on the testpiece had suffered a little in 17 years storage. We are now aiming to carry this work on farther, because it does seem to throw a new light on the concept of fatigue developing in material previously damaged by plastic deformation, that is, slip.

Fatigue from the Metallurgist's Viewpoint

by

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Introduction

The term "fatigue" is given to the progressive failure of a metal under the repeated application of a cycle of stress. Although investigators have been working for more than a hundred years to determine the mechanism by which fatigue failure occurs, the problem is still not fully understood. The simple slip theories of Ewing and Rosenhain[1] in 1899, Ewing and Humphrey[2] in 1903 and of Gough and his collaborators[3, 4, 5] from 1923 to 1933 appear now to be only partially true, according to some of the modern theories[6, 7, 8]. These suggest that other factors such as polygonisation, deformation bands and crystallite formations, all play an important part.

Metallurgists and engineers are only too well acquainted with the characteristics of fatigue failure. Fenner[9] has described these in the following manner:—

> "The process of failure by fatigue may be said to take place in three stages. In the first stage, cycles of stress are applied to the material without any visible damage occurring. After a number of applications of load, a minute crack commences at a nucleus, usually at an exterior surface, and spreads from this point through the stressed section. The development of the crack from its beginning may extend over several millions of stress cycles, though it is usually of somewhat shorter duration. Finally, the reduced effective area of the section becomes insufficient to support the load to which it is subjected and complete failure occurs suddenly, often with but little of the distortion normally associated with ductile materials. The appearance of the surface of the fracture often contains signs which are identifiable as evidence of fatigue. The area to which the crack had extended before final rupture is often sharply delineated, its boundary in the material being smoothly curved. The surface of this area is smooth in texture owing to the interaction between the two sides of the crack during its growth and is also normally less subject to major irregularities than the remainder of the fracture. The crack area is sometimes characterised by a series of concentric arcs, circular or elliptical, whose common centre marks the position of the 'fatigue nucleus' at which fracture commenced. The pronounced similarity to the markings on a sea-shell has led to the use of the term 'conchoidal' to describe this type of fracture."

The majority of fatigue failures can be attributed to what may be termed engineering faults, in that failure is brought about by the exploitation of stress concentrations arising from inadequate design, poor execution, and lack of inspection. The metallurgist can be blamed for occasional failures which result from the provision of sub-standard material, unsuitable surface coatings and faulty heat treatment. Some of the metallurgical factors which affect the life of a component either beneficially or otherwise are:—

1. Structural discontinuities.
2. Grain flow.
3. Nitriding.
4. Cyaniding.
5. Carburising.
6. Decarburisation.
7. Surface hardening by heating.
8. Cold work.
9. Surface coatings.
10. Influence of temperature.
11. Influence of environment.

1. Structural Discontinuities

Inclusions

The published work on the effect of inclusions on the fatigue properties of wrought steels has been mainly on carbon and alloy steels of up to 70 tons/in.^{2} Ultimate Tensile Stress (U.T.S.). These tests have shown that the fatigue properties of these materials are not affected to any appreciable extent. However, Johnson[10] has stated that a reduction of up to 20 per cent. occurred in the reversed bending fatigue limit of a 67 tons/in.^{2} U.T.S. Ni-Cr steel as the number and size of the inclusions increased.

In some unpublished work at Bristol, Frith has shown that a wrought steel of 80 to 100 tons/in.^{2} U.T.S., regarded as being very clean by the standard metallographic examination, possessed considerably reduced longitudinal fatigue properties when only a few inclusions of a certain type were present. Work is in hand on this problem at Bristol and a report is being prepared for publication.

Cast Iron

It might be expected that the numerous graphite flakes present in cast iron would act as internal notches and that the fatigue limit of polished cast iron bars would therefore be lower in relation to the tensile strength than in steel, but this ratio is very nearly as high as that in steel. Furthermore, the notched fatigue limit of cast iron is generally higher than that of a steel of the same tensile strength, despite the lack of ductility in cast iron. It is true, however, that notch sensitivity gradually increases with increasing tensile strength, but even in the high strength cast irons the sensitivity is relatively low, as shown by the work of Kommers[11] in Table I.

TABLE I

EFFECT OF SQUARE NOTCHES ON THE FATIGUE LIMIT OF CAST IRONS—KOMMERS

U.T.S. TONS/□"	FATIGUE LIMIT (TONS/□")		% DECREASE DUE TO NOTCH
	SMOOTH	SQUARE NOTCH	
9·0	4·15	4·15	0 %
10·5	5·25	5·0	5 %
13·5	6·7	6·15	8 %
15·0	9·85	7·0	29 %
6·5	8·5	7·3	16 %
19·0	10·75	8·5	21 %

2. Grain Flow

The test results in Table II indicate that when the fatigue stress occurs transversely to the direction of the grain flow, reductions in the reversed bending fatigue limit may occur, the amount of reduction increasing apparently with increasing tensile strength of the material. Aitchison and Johnson[12], Schmidt[13], Rossing[14] and others[15] have shown that the average reduction in the reversed bending fatigue limit for steels of up to 60 tons/in.^{2} U.T.S. is about 15 per cent. when tested in the transverse direction. The results themselves are rather variable, probably because of variations in the amount of reduction from ingot to bar, the position of the test piece in relation to the original ingot macrostructure and the type or condition of the material.

Only a few tests have been done on wrought steels heat-treated to higher ultimate tensile strength values. Woolman and Jaques[15] have reported a 27 per cent. reduction in the reversed bending fatigue limit of a 4¼ Ni-Cr steel (EN.30) heat-treated to 118 tons/in.^{2} U.T.S. when tested in the transverse direction and a 16 per cent. reduction on a Mn-Mo steel (EN.16) heat-treated to 89 tons/in.^{2} U.T.S. when similarly tested.

The results indicate that as the tensile strength increases, the percentage reduction in the reversed bending fatigue limit due to testing in the transverse direction also increases. Therefore, from a fatigue point of view, the attainment of a satisfactory forging grain flow becomes even more essential as the ultimate tensile strength is raised above 60 tons/in.^{2}.

It is probable that the reversed bending fatigue properties of non-ferrous alloys will be decreased when tested in the transverse instead of in the longitudinal direction. There is little published data on this subject but Schröter[16] cites Brenner and Kostron as stating that the endurance limit of an aluminium-copper-magnesium alloy when tested in the transverse direction was some 17 per cent. below that in the longitudinal direction.

3. Nitriding

Nitriding is defined as:—"A process of case-hardening in which a ferrous alloy, usually of special composition, is heated in an atmosphere of ammonia or in contact with nitrogenous material to produce surface hardening by the absorption of nitrogen without the necessity of quenching."

Table III shows the results of fatigue tests made by Frith[17], Sutton[18], Bardgett[19], Johnson and Oberg[20] and others[21], [22], [23]. It can be seen that nitriding does

TABLE II

THE EFFECT OF "GRAIN" DIRECTION ON THE REVERSED BENDING FATIGUE LIMIT OF VARIOUS STEELS.

STEEL		U.T.S. TONS/□"	FATIGUE LIMIT WITH GRAIN LONGITUDINAL TONS/□"	% REDUCTION IN FATIGUE LIMIT WITH TRANSVERSE GRAIN TONS/□"	REFERENCES
ARMCO IRON		21·5	±11·8	0 %	AITCHISON AND JOHNSON [12]
WROUGHT IRON		24·4	±10·0	5 %	
3·5 % Ni — Cr STEEL		65·0	±31·8	7%	
1·0% Cr — Ni — Mo STEEL BAR	RIM	51	±24·0	29%	RÖSSING [14]
	CENTRE	50	±19·0	16 %	
1·0% Cr — Mo STEEL BLOCK	EDGE	52	±30·0	7 %	
	CENTRE	50	±27·0	2 %	
1·0% Cr - Ni-Mo STEEL CONNECTING ROD		65·5	±27·0	13%	
0·8% Ni STEEL SHAFT RIM	AS FORGED	38	±16·0	25%	
	NORMALISED	40	±17·0	19%	
4·25% Ni - Cr STEEL (SOFTENED)		59	±30·0	17%	WOOLMAN AND JAQUES [15]
4·5 % Ni - Cr - Mo STEEL (O Q)		89	±44·0	16%	
4·25% Ni - Cr STEEL (A H)		118	±48·8	27%	

not affect the unnotched fatigue properties under axial loading to any appreciable extent, but it is very effective under reversed bending conditions, giving increases in the fatigue limit of up to 50 per cent. Frith[17] has also shown that the reversed torsional fatigue limit of an unnotched Cr-Mo steel test piece can be increased by 20 per cent. after nitriding.

The effectiveness of the treatment on the axial, reversed bending and reversed torsional fatigue limits of notched test pieces is far greater. Bardgett[19] has reported an increase of 142 per cent. in the axial fatigue limit on a 90 tons/in.² U.T.S. Cr-Mo-V steel and an increase of 189 per cent. in the reversed bending fatigue limit of an 85 tons/in.² U.T.S. Cr-Mo steel. Frith[17] has also shown that the reversed bending and reversed torsional fatigue limits are increased by 90 and 88 per cent. respectively on a 62 tons/in.² U.T.S. Cr-Mo steel. In other words, the nitriding has minimised the stress concentration effect on these materials.

It has been noted that in the absence of stress concentration the fracture of nitrided test pieces subjected to reversed bending, or reversed torsional stresses generally begins near the junction between case and core. It is not definitely known what causes the fracture to start below the surface but Frith[17] is of the opinion that it may be due to one or both of the following reasons:—

(*i*) Compressive stresses in the nitrided case.

(*ii*) The fatigue properties of the nitrided case being higher than those of the core.

The more likely reason is the first, the compressive stress in the nitrided case being balanced by a tensile stress in the core. The compressive stress in the case is regarded as being beneficial from a fatigue point of view for reversed bending and reversed torsional stresses, but the balancing tensile stress in the core may be detrimental on test pieces or components of small cross-sectional area. This stress in the core may also be detrimental for axial tension-compression test pieces, or components of small cross-sectional area where the ratio of case thickness to core thickness is high. Therefore, before nitriding is used to improve the fatigue properties of an engine or airframe component, the type of stress imposed in service and the dimensions of the components must be considered.

For suitable components, nitriding can be used to great advantage and work done at Bristol has shown that a nitriding time of 12 hours is generally sufficient. After nitriding for 12 hours, final sizing can be done by lapping or honing, instead of by grinding which is needed if longer nitriding periods are used. Grinding is necessary to obtain the maximum hardness on the thicker nitrided layers and, unless very carefully done, is liable to produce surface cracks on the component, with a consequent drastic reduction in the fatigue properties.

Since the process is done at temperatures between 500° and 600°C., depending upon the alloy steel used, the recovery hardness of the case and core does not decrease until the component is heated above the nitriding temperature. Nitrided components therefore can be used up to temperatures far higher than carburised components, for which the hardness of case and core begins to decrease when heated to temperatures above 200°C. Nitriding has also been found to be beneficial for use under corrosion-fatigue conditions as shown by Bardgett[19]. This is especially true when the surface has not been ground after the nitriding operation.

4. Cyaniding

Cyaniding is defined as:—"A process of case-hardening a ferrous alloy by heating in a molten cyanide, thus causing the alloy to absorb carbon and nitrogen

TYPE OF TEST	STEEL	U.T.S. TONS/□"	FATIGUE LIMIT UNNITRIDED TONS/□"	% IMPROVEMENT IN FATIGUE LIMIT DUE TO NITRIDING	REFERENCES
AXIAL—UNNOTCHED	Cr-Mo-V	90	± 40·0	−5%	BARDGETT [19]
AXIAL—NOTCHED			± 10·5	+142%	
REVERSED BENDING UNNOTCHED	Cr-Mo	50	± 27·0	+50%	HENGSTENBERG AND MAILANDER [21]
		62	± 33·0	+23%	FRITH [17]
		85	± 47·5	+20%	BARDGETT [19]
	Cr-Al	51	± 27·0	+31%	HENGSTENBERG AND MAILANDER [21]
		61	± 32·0	+33%	
	Cr-Mo-V	90	± 41·5	+24%	BARDGETT [19]
	Cr-Al-Mo	58	± 31·0	+29%	HENGSTENBERG AND MAILANDER [21]
	Cr-Al-Ni	70	± 35·0	+24%	
	Cr-Ni-Mo-Al	71	± 34·0	+10%	MOCHEL [22]
REVERSED BENDING NOTCHED	Cr-Mo	62	± 14·5	+90%	FRITH [17]
		85	± 17·8	+189%	BARDGETT [19]
	Cr-Al	45	± 13·0	+188%	MAILÄNDER [23]
	Cr-Mo-V	90	± 18·75	+170%	BARDGETT [19]
	Cr-Al-Mo	62	± 18·0	+116%	MAILÄNDER [23]
TORSION—UNNOTCHED	Cr-Mo	62	± 19·9	+20%	FRITH [17]
TORSION—NOTCHED			± 10·5	+88%	

TABLE III

THE EFFECT OF NITRIDING ON THE FATIGUE LIMIT OF VARIOUS STEELS.

simultaneously." Cyaniding is usually followed by quenching to obtain a hard case and is used to produce hard thin cases on components subjected to light loads.

There is little published data on the effect of cyaniding on the fatigue properties of steels, but the work of Lea[24] on mild steel test pieces has indicated that an improvement in the reversed bending fatigue limit of some 97 per cent. can result after this treatment. Hankins and his co-workers[25], [26] in tests on various steels in either the hot worked and decarburised, or polished and decarburised condition have shown that cyaniding resulted in a 5 to 60 per cent. improvement in the reversed bending fatigue limit in the first case, and 38 to 74 per cent. improvement in the latter. These results, together with those of Lea[24], are shown in Table IV.

5. Carburising

Carburising is defined as:—"A process that introduces carbon into a solid ferrous alloy by heating the metal in contact with a carbonaceous material—solid, liquid or gaseous—to a temperature above the transformation range and holding at that temperature, followed by quenching to produce a hardened case."

The work of Wiegand[27], Wiegand and Scheinost[28] and Woodvine[29], on carburised unnotched and notched test pieces has shown that considerable improvements

STEEL	U.T.S. TONS/□"	SURFACE CONDITION	FATIGUE LIMIT BEFORE CYANIDING TONS/□"	% IMPROVEMENT IN FATIGUE LIMIT DUE TO CYANIDING	REFERENCES
0·16%C	28·8	HIGHLY POLISHED	± 15·0	+ 97%	LEA [24]
SILICO-MANGANESE	92	AS ROLLED (DECARBURISED SURFACE)	± 25·0	+ 60%	HANKINS BECKER AND MILLS [26]
0·2%C	31·5	AS FORGED (DECARBURISED SURFACE)	± 12·0	+ 58%	
0·4%C	43		± 16·5	+ 15%	
3%Ni	55		± 15·0	+ 27%	
Ni-Cr	63		± 18·0	+ 5·5%	
SILICO-MANGANESE	92	SURFACE POLISHED THEN FURNACE HEAT TREATED (DECARBURISED SURFACE)	± 27·0	+ 74%	HANKINS AND BECKER [25]
Cr-V	80		± 32·0	+ 38%	

TABLE IV

THE EFFECT OF CYANIDING ON THE REVERSED BENDING FATIGUE LIMIT OF VARIOUS STEELS.

in the reversed bending and reversed torsional fatigue properties can result, depending upon the material and treatment used, as shown in Table V. Unnotched test pieces gave a 32 to 105 per cent. improvement in the reversed bending fatigue limit, and notched test pieces 82 to 230 per cent. It should be noted that test pieces with a transverse hole drilled before and after carburising gave respectively a 29 per cent. improvement and a 38 per cent. reduction in the reversed bending fatigue limit. Similar test pieces subjected to reversed torsional stresses gave respectively a 142 per cent. improvement and a 17 per cent. reduction in the fatigue limit. These results indicate that the drilling of holes, or any other operation which is likely to produce areas of high stress concentration, should be done before carburising, if an even greater reduction in the notched fatigue properties is to be avoided.

The effect on the fatigue properties due to carburising wrought steels is similar to that obtained by nitriding, as both treatments result in the production of a compressive stress in the case, and a tensile stress in the core. The decision whether to use nitriding or carburising for a component depends upon the type of steel, the depth of case required, the temperature attained by the component during service and the distortion permitted due to case-hardening. If the service temperature is above 200° C., nitriding is to be preferred, but if a case depth greater than 0·015 in. is wanted, carburising would be better. Distortion is generally greater for carburising than for nitriding and a final grinding operation is often necessary after carburising. This operation needs to be done carefully as Almen[30] has shown that considerable tensile stresses may be left after grinding in a thin surface layer which before possessed a compressive stress. Carelessness in this respect therefore can lead to a reduction in the fatigue properties of a carburised component.

6. Decarburisation

Decarburisation is defined as:—"The unintentional production of a weak ferritic layer on the surface of a steel component resulting from the removal of carbon by oxidation during heat treatment, rolling, forging and so on."

Since the surfaces carry the maximum stress under fatigue conditions even when the loading is ostensibly axial, it can be expected that the presence of a soft weak outer layer will lead to a reduction in the fatigue properties of a steel. In general, the reduction becomes more marked as the tensile strength and notch sensitivity of the core material increases, as shown by Kenneford and Ellis[31] and Jackson and Pochapsky[32]. In view of this, it is not surprising that the high tensile wire used for the manufacture of springs is so susceptible to the presence of decarburisation. Hankins and Becker[25] have shown that the reversed bending fatigue limits of Cr-V and Si-Mn spring steels were reduced from ±42 and ±46 tons/in.² respectively to ±20 tons/in.² when heavily decarburised, a reduction of some 55 per cent.

In many cases it is relatively easy to remove the decarburised layer from heat-treated components by machining, but with spring wire it is often impossible, or uneconomic, to remove the final traces. Spring wire components have a considerable area of stressed surface and are particularly susceptible to the presence of even isolated points of weakness. The general fatigue properties of the wire are thus reduced to those of the weak spots and early fatigue failure is inevitable under the

TABLE V

THE EFFECT OF CARBURISING ON THE FATIGUE LIMIT OF VARIOUS STEELS.

TYPE OF TEST	STEEL	U.T.S. TONS/□"	FATIGUE LIMIT UNCARBURISED TONS/□"	% IMPROVEMENT DUE TO CARBURISING	REFERENCES
REVERSED BENDING. UNNOTCHED	0·16% C	50	± 16·5	+ 88%	WIEGAND[27]
	1·0% Cr	60	± 26·0	+ 105%	
	1·2% Cr . 1·0% Mn . 0·25% Mo	65	± 29·0	+ 72%	
	2% C . 0·2% Mo . 2·0% Ni	80	± 38·0	+ 32%	
	3·5% Ni	62	± 29·0	+ 85%	WOODVINE[29]
	5·0% Ni	72	± 34·0	+ 62%	
REVERSED BENDING. NOTCHED	0·16% C	30	± 14·0	+ 82%	WIEGAND[27]
	1·0% Cr	60	± 12·7	+ 95%	
	1·2% Cr . 1·0% Mn . 0·25% Mo	65	± 12·7	+ 230%	
	2·0% Cr . 0·2% Mo . 2·0% Ni	80	± 10·2	+ 219%	
REVERSED BENDING. WITH TRAVERSE HOLE	2·0% Cr . 2% Ni . 0·25% Mo	80	± 21·6	− 38% HOLE DRILLED AFTER CARBURISING + 29% HOLE DRILLED BEFORE CARBURISING	WIEGAND AND SCHEINOST[28]
TORSION. WITH TRAVERSE HOLE			± 7·6	− 17% HOLE DRILLED AFTER CARBURSING + 142% HOLE DRILLED BEFORE CARBURISING	

very high loads they are expected to carry. The work of Hankins and Becker[25] and Hankins, Becker and Mills[26], on the effect of even minor amounts of decarburisation on the fatigue properties of Si-Mn steel has clearly illustrated this point. It is important therefore to note that the fatigue properties of decarburised spring wire can be brought up to those of polished wire which has not been decarburised, by the simple and inexpensive process of shot peening the surfaces. The theory, applications and the beneficial results obtained in this manner are dealt with later.

7. Surface Hardening by Heating

Information is scanty on the improvements in the fatigue properties brought about by the flame or induction hardening of surfaces, although both treatments are being used increasingly to provide wear-resistant surfaces.

Flame Hardening

Flame hardening is defined as:—"A surface-hardening process applied to steel in which the surface is heated to a temperature above the transformation range by means of a flame, followed immediately by quenching."

Zimmerman[33] has reported that the process produced an 80 per cent. increase in the reversed bending fatigue limit of a 0·5 per cent. C-Mn steel test piece with a flame hardened fillet and 190 per cent. increase when the whole test piece was treated. Wiegand[27] has also reported increases of 60 to 88 per cent. on testing unnotched medium carbon steel test pieces in reversed bending and an 18 per cent. increase on testing in reversed torsion. The increase in the fatigue limit arising from this treatment is probably caused through the compressive stresses present in the hardened surface layer, the degree of change depending on the condition and composition of the original material. It must be remembered, however, that thermal cracking may take place as a result of this treatment, with a consequent reduction in the reversed bending fatigue limit due to stress concentration in the cracks, as found by Horger and Cantley[34] in work on a 0·5 per cent. C-Mn steel.

Induction Hardening

Induction hardening is defined as:—"A surface-hardening process applied to steel in which the surface is heated to a temperature above the transformation range by high frequency alternating currents followed immediately by quenching."

The published data on induction hardening and its effect upon the fatigue properties of steel is both scarce and contradictory. Johnson[35] has reported that induction hardening of a C-Mn steel shaft containing a transverse oil hole reduced the reversed torsional fatigue limit by 25 per cent. It was found, however, that this reduction could be avoided by shot peening the induction hardened surfaces to produce a compressive stress. Where this cannot be done, Johnson[35] considers that the hardening should not be carried into areas of high stress concentration. Williams and Brown[36], on the other hand, tested a 3 per cent. Ni-Cr-Mo steel crankshaft with induction-hardened journals, the fillets of which had not been hardened, and reported a 42 per cent. reduction in the reversed bending fatigue limit. As would be expected under these conditions, the failure occurred not in the hardened zone but adjacent to it in one of the unhardened fillets. No metallurgical or hardness changes were noted in the fractured area, but an independent x-ray examination made on the component indicated the presence of residual stresses ranging from 5 to 16 tons/in.^{2} in the area adjacent to the hardened zone, no indication being given as to the nature of these stresses. In view of the presence of these residual stresses in an area which is normally highly stressed under reversed bending fatigue conditions, Williams and Brown[36] are of the opinion that better results would have been obtained if the hardening had been carried right round the fillet into the crank web. In this manner any residual stresses in the area adjacent to the hardened zone would then occur in a part of the component which is lightly stressed under normal conditions.

Surface hardening by either flame or induction methods should not be used on components running at elevated temperatures, due to the softening of the hardened surface layers, the limiting temperature being 200°C., as for carburising.

8. Cold Work

Cold working is defined as:—"A process in which the metal is permanently deformed by working it at a temperature below the recrystallisation range."

This process results in an increase in the tensile strength and fatigue properties of the material and, in its notch sensitivity. While the increase in the fatigue properties depends upon an increase in the degree of working, severe cold working can lead to the formation of surface cracks with a consequent drastic reduction in the fatigue limit. This process should not be used on steel components running at temperatures above 250°C., because the increasing temperature produces progressive softening of the distorted material and a consequent lowering of the beneficial compressive stress. Cold working can be achieved either by rolling, drawing, surface rolling or shot peening. The harmful effect on fatigue properties caused by the cold straightening of bent components is referred to later.

8.1. ROLLING AND DRAWING

Moore and Kommers[37] have stated that a cold drawn mild steel had a reversed bending fatigue limit some 46 per cent. greater than that of the same steel in the hot rolled condition, while Dolan and Price[38] have reported that a cold drawn leaded 0·5 per cent. C steel had a reversed bending fatigue limit some 16 per cent. greater than in the normalised condition. They have also shown that cold work increases the notch sensitivity. Notching the cold drawn leaded steel test pieces caused a 60 per cent. reduction in the reversed bending fatigue limit as against a 37 per cent. reduction in the normalised and notched condition.

8.2. SURFACE ROLLING

The fatigue properties of a steel component may be increased by cold working the surface with small rollers under pressure. The process has been in use for many years as a means of hardening shaft surfaces and more recently it has been used successfully for the rolling of threads, fillets and keyways. Some of the improvements obtained in the fatigue properties of various steels in this manner are given in Table VI. Dinner and Felix[39] found in work on a 0·2 per cent. C-Mn steel that test pieces with rolled threads possessed a fatigue limit some 58 per cent. greater than those with ground threads under axial loading, while Love[40] cites Burnheim as reporting that the reversed bending fatigue limit of two notched steels (0·25 per cent. C-Mn and 0·13 per cent. V-Mo) increased by 88 and 46 per cent. respectively after rolling. Horger[30] cites various investigators as having found that rolling of fillets increased the reversed bending fatigue limit by 30 to 69 per cent. depending upon the material, while Koch[41] found an increase of about 20 per cent. in the reversed bending fatigue limit resulting from the cold rolling of keyways in material of 20 and 30 tons/in.^{2} U.T.S.

There is little data on the effect of surface rolling non-ferrous alloys but Sachs[41] has reported that on the basis of 10^7 cycles, the reversed bending endurance limit of a three-inch diameter 8 per cent. Al-Mg alloy test piece was increased by 50 per cent. after this treatment. This process has actually been used to improve the fatigue properties of magnesium alloy propellers.

8.3. SHOT PEENING

Shot peening is defined as:—"A process whereby the surfaces of a component are hardened by the high speed hammering action of many small metal balls." The process has its foundation in the theory that work hardening produces a compressive stress on the surface of the component, which results in an increase in the fatigue properties of the material, as shown by the results in Table VII. Much work has been done on this subject by Almen and Moore and they have contributed to a book entitled "Shot Peening"[43].

Tests by Moore[43] and others[44], [45], [46] on steel specimens initially polished have shown that shot peening resulted in improvements in the reversed bending fatigue properties which varied with the tensile strength of the material. These improvements ranged from some 15 per cent. for steels of up to 40 tons/in.^{2} U.T.S. to a negligible improvement, or slight reduction, for the 80 to 100 tons/in.^{2} U.T.S. steels.

In many cases, however, it is impracticable or uneconomic to provide a high standard of surface finish and many components have to be put into service either in the "as heat treated" or "as ground" condition. Decarburisation, scratches, tool marks and residual tensile stresses induced by grinding may be present on these surfaces, with the result that early fatigue failure is often inevitable when the material has a high ultimate tensile strength. It is claimed that shot peening will remove these harmful effects and raise the fatigue limit until it approaches that obtained on polished surfaces, and at a fraction of the cost. Moore[43] has shown in a series of tests on a wide variety of steels, that improvements in the reversed bending and reversed torsional fatigue limits ranged from 23 to 100 per cent. when shot peened surfaces were compared with the "as rolled," "as drawn" and "as heat treated" surfaces. For non-ferrous alloys Wiegand[46] has stated that on the basis of 10^7 cycles, the improvements in the reverse bending endurance limits gained by shot peening the fine-turned surfaces of Duralumin, an Mg alloy (A.Z.31), and an Elektron casting (AZG) were respectively 16, 26 and 24 per cent. According to Moore[43] a phosphor-bronze alloy in the form of "as rolled" sheet showed, on the basis of 10^7 cycles, an improvement of 54 per cent. in the reverse bending endurance limit after shot peening.

TABLE VI

THE EFFECT OF SURFACE ROLLING ON THE FATIGUE LIMIT OF VARIOUS STEELS.

TYPE OF TEST	STEEL	U.T.S. TONS/□"	DIA OF T/P	FATIGUE LIMIT BEFORE SURFACE ROLLING TONS/□"	% CHANGE IN FATIGUE LIMIT DUE TO ROLLING	REFERENCES	
REVERSED BENDING. UNNOTCHED	0·45% C — Mn (S.A.E. 1045)	40	0·3"	± 17·9	+30%	HORGER AND NEIFERT[42]	
			1·0"	± 16·1	+20%		
			7·0"	± 7·8	+>86%		
AXIAL. NOTCHED	0·2%C. 0·5%Mn	30	1·0"	± 11·7	+58%	DINNER AND FELIX[39]	
REVERSED BENDING. NOTCHED	0·25%C. 1·17%Mn 0·81%Cr	—	0·275"	± 16·5	+88%	BURNHEIM	GIVEN BY LOVE[40]
	0·13%V. 0·07% Mo	—	0·275"	± 16·5	+46%		
REVERSED BENDING FILLET	0·12%C	26	0·709"	± 9·5	+57%	THUM AND BAUTZ	GIVEN BY HORGER[30]
	0·36%C	38	0·67	± 10·4	+56%	THUM AND BRUDER	
	0·56%C	54	0·59"	± 17·8	+30%		
	0·38%C. 1·5%Ni	59	0·67"	± 13·3	+69%		
	0·53%C. 0·95%Mn	46	5·25"	± 8·3	+30%	HORGER BUCKWALTER AND NEIFERT	

In production it is essential that uniform fatigue properties should be maintained on large numbers of components and careful control procedures would appear to be necessary if shot peening is to be used as a method of obtaining them. In this connection, Zimmerli[47] tested some spring wire and noted that too coarse a grade of shot resulted in over-peening, while Bardgett and Gartside[48] have shown that prolonged peening of an 18:8 nickel-chromium steel caused a reduction in the fatigue properties below the optimum.

Shot peening the surface of a steel also improves the corrosion fatigue resistance, according to Almen[49]. This improvement is apparently due to the fact that corrosion pits do not have their usual damaging effect on the material until they finally penetrate the compressively stressed surface layer formed by the process.

8.4. COLD STRAIGHTENING

Components sometimes have to be cold-straightened to remove the slight distortion arising from heat treatment. This process can produce residual surface tensile stresses which may be detrimental. Schmidt[50], for example, has found that crankshafts straightened at temperatures up to 180°C. had a reversed bending fatigue limit some 20 per cent. less than that of shafts in the unstraightened condition. Horger and Lipson[51] have reported similar results when making reversed bending fatigue tests on automobile rear axles and state that in one instance residual tensile stresses of about 54 tons/in.^{2} were found after straightening. The effect on notched test pieces is reputed to be even worse, some 50 per cent. reduction in the reversed bending fatigue limit being quoted in unpublished work cited by Love[40].

These reductions, as already stated, are caused by the formation of residual tensile stresses and not by cracks produced during the straightening process. However, cold-straightening of nitrided or carburised components can easily lead to the production of small cracks in the hardened case, thus producing even greater reductions in fatigue properties than those resulting from the presence of residual tensile stresses.

9. Surface Coatings

From a performance point of view a coating applied to the surfaces of a component is often required to increase its wear or corrosion resistance. The coating may fulfil these requirements but at the same time may affect the fatigue properties of the component. Some of the more usual methods of surface coating and their effect under normal and corrosive conditions are now discussed.

9.1. ELECTRO-PLATING

The effect of electro-plating on the fatigue properties of wrought ferrous and non-ferrous alloys can be divided into two sections. The first includes those metals which do not affect the fatigue properties at atmospheric temperture to any appreciable extent, such as copper, cadmium, lead, tin and zinc, and the second deals with nickel and chromium which can cause considerable

TABLE VII

THE EFFECT OF SHOT PEENING ON THE FATIGUE OR ENDURANCE LIMIT OF VARIOUS FERROUS AND NON-FERROUS MATERIALS

TYPE OF TEST	MATERIAL	U.T.S. TONS/□"	SURFACE CONDITION	FATIGUE OR ENDURANCE LIMIT BEFORE SHOT PEENING TONS/□"	% IMPROVEMENT DUE TO SHOT PEENING	REFERENCES
REVERSED BENDING	ARMCO IRON	20	MACHINED AND POLISHED	±12	+1%	LESSELLS AND MURRAY[44]
	C-Mn STEEL (S.A.E. 1045)	40		±13·8	3 - 19% (VARIOUS TREATMENTS)	HORGER AND NEIFERT[42]
	Ni-Cr-Mo STEEL (S11)	575		±29	+2% (COARSE SHOT) 0% (FINE SHOT)	ROBERTS AND GRAYSHON[45]
	2%Cr-Ni-Mo STEEL	70		±35·6	+2%	WIEGAND[46]
	C-Mn STEEL (S.A.E. 9260)	90		±49	-2%	LESSELLS AND AND MURRAY[44]
	DURALUMIN (AL-ALLOY)	27		±7·6 *	+16·5%	WIEGAND[46]
	AZ.31.(Mg ALLOY)	18		±6 *	+26%	
	C-Mn STEEL (S.A.E. 1020)	30	AS ROLLED	±12·5	+34%	MOORE[43]
	Ni-Cr-Mo STEEL CARBURISED & H.T.	—	AS HEAT TREATED	±26	+23%	
	NE 9420 STEEL (CARBURISED)	—	AS CARBURISED	±44·5	+53%	
	PHOSPHOR BRONZE SHEET	—	AS ROLLED	±14 *	+54%	
TORSION	18/8 STEEL WIRE	—	AS DRAWN	±29	+69%	
	18/8 STEEL BAR	—	AS ROLLED	±205	+100%	

* ENDURANCE LIMITS ON A BASIS OF 10 CYCLES

reductions unless care is taken in their application. It is not considered necessary to enlarge upon the metals contained in the first section, so the remarks on the effect of electro-plating on the fatigue properties of ferrous and non-ferrous materials will be confined to nickel and chromium.

(a) Nickel Plating

The test results in Table VIII illustrate the effect of nicket plating on the fatigue properties of some ferrous and non-ferrous alloys. Love[40] cites Lea as having found that nickel plating produced a 37 per cent. reduction in the axial fatigue limit of mild steel and Barklie and Davies as reporting that the reduction in the reversed bending fatigue limit resulting from nickel plating mild steel increased from 7 to 40 per cent., with increases in the internal stress present in the plating. Barklie and Davies have also stated that, for a given plating condition, the reduction increased from 3 to 44 per cent. on increasing the thickness deposited from 0·0005 in. to 0·0035 in. This trend has been confirmed by some unpublished work by Frith done at Bristol on a Ni-Cr steel (S.11) with a U.T.S. of 64 tons/in.^{2} Plating to a thickness of 0·0015 in. caused a reduction in the reversed bending fatigue limit of 38 per cent. which was further reduced to 57 per cent. on increasing the thickness to 0·015 in. Love[40] also quotes Barklie and Davies as having demonstrated that when the nickel was removed, the reversed bending fatigue limit was restored to the value of the unplated test piece, indicating that the damaging effect was confined to the deposit.

The value of the stresses present in the plating tend to vary according to the conditions of plating and the type of bath used, as shown by Brenner and Jennings[52]. They reported that residual tensile stresses ranging from 0 to 36 tons/in.^{2} were found in nickel deposited from a variety of baths. The work of Hothersall[53] on a steel of 40 to 50 tons/in.^{2} U.T.S. has shown that plating with little or no internal stress has a very small effect upon the reversed bending fatigue limit, highly stressed plating may produce a 50 per cent. reduction, and normally stressed deposits result in a 25 per cent. reduction.

On the plating of non-ferrous alloys, Frith in some unpublished work at Bristol on an aluminium alloy (RR.56) and a 0·5 per cent. Cr-Cu alloy (BACE.197) has shown that the effect upon the reversed bending endurance limit is negligible in both cases. On the basis of 10^8 cycles, the endurance limit was reduced by 6 per cent. in the first instance and increased by 2 per cent. in the second.

It can be concluded from the findings of these investigators that, where possible, the use of nickel plate should be avoided when a steel component is subjected to alternating loads. If, however, nickel plated surfaces are essential for the successful operation of the component, then particular care should be taken to ensure that the process used will deposit metal with a minimum tensile stress. Furthermore, it would appear to be good practice to use the minimum thickness of deposit conducive to satisfactory performance.

Nickel plating is particularly useful for parts subjected to corrosion fatigue conditions. This is shown by Cazaud[54], who demonstrated that the fatigue properties of plated mild steel test pieces were not reduced on testing in fresh water and by Wilson[55], who reported that the reversed bending endurance limit in salt spray of a mild steel plated with 0·005 in. of nickel was some 67 per cent. greater than that of the material in the unplated condition.

(b) Chromium Plating

Many investigators have shown that, depending upon the conditions, chromium plating can cause a reduction of 50 per cent. or more in the reversed bending fatigue

TABLE VIII

THE EFFECT OF NICKEL PLATING ON THE FATIGUE OR ENDURANCE LIMIT OF VARIOUS MATERIALS.

TYPE OF TEST	MATERIAL	U.T.S. TONS/□"	FATIGUE LIMIT BEFORE PLATING TONS/□"	% CHANGE IN FATIGUE OR ENDURANCE LIMIT DUE TO PLATING	REFERENCES	
AXIAL	0·14%C STEEL	—	± 14·5	− 37%	LEA	GIVEN BY LOVE[40]
REVERSED BENDING	0·09%C STEEL	—	±15·0	− 7% (LOW INTERNAL STRESS)	BARKLIE AND DAVIES	
				− 30% (HIGH INTERNAL STRESS)		
	0·14%C STEEL	—	± 16·0	− 3% (0·0005" PLATE) − 44% (0·0035" PLATE)		
	ARMCO IRON	20	± 13·25	+ 8·7% (0·004" PLATE)	FRITH +	
	S11(Ni-Cr STEEL)	64	± 35·25	− 38% (0·0015" PLATE) − 57% (0·015" PLATE)		
	RR56 (Al ALLOY)	26	± 8·1 *	− 6% (0·005" PLATE)		
	BACE 197 (Cu-Cr ALLOY) (TESTED AT 300°C)	7	± 3·6 *	+ 2% (0·010" PLATE)		

* ENDURANCE LIMIT ON BASIS OF 10^8 CYCLES AT 2,000 C.P.M.

+ UNPUBLISHED WORK.

limit of steels, especially those with high ultimate tensile strengths, as shown in Table IX.

According to Mehr, Oberg and Teres[56], both an increase in deposit thickness and changes in the conditions of plating can result in a serious reduction in the reversed bending fatigue limit of a 2 per cent. Ni-Cr-Mo steel. They have quoted reductions of 8 to 50 per cent., depending upon the thickness of the plating, temperature and current density used. Wiegand[57] also reported an increasing reduction in the reversed bending fatigue limit of a 2 per cent. Cr-Mo steel, with increases in the thickness of plating.

This data has been provided from tests made on small standard fatigue test pieces and the good agreement shown by Williams and Brown[36] on an actual component is interesting. They plated the journals of a 3 per cent. Ni-Cr-Mo steel crankshaft to a depth of 0·008 in. and found a 40 per cent. reduction in the reversed bending fatigue limit. During the 1939-45 War the author gained some experience in the salvaging of worn aero-engine crankshafts by chromium plating. No fatigue failures occurred during the subsequent running life of these salvaged components as a result of this treatment. Great care, however, was taken to avoid plating around the radii of fillets or other areas subjected to high fluctuating stresses and to ensure that no stress raisers existed at the plating run-out.

The possibility of minimising the harmful effect of chromium plating by a post-plating heat treatment has been investigated by a number of workers. Mehr, Oberg and Teres[56] and Wiegand and Scheinhost[58] have shown that heating plated Ni-Cr-Mo steel test pieces up to 205°C. caused an even greater reduction in the fatigue properties. Logan[59], however, has demonstrated that plated test pieces heat-treated for one hour at 440°C. possessed fatigue properties significantly higher than those of the "as plated" test pieces, but still inferior to the unplated material. This treatment does not appear to be detrimental to the wear resistance of the plate and could well be used on steels previously tempered to at least 500°C.

Carburising before plating is not advantageous since Wiegand[46] has reported a 60 per cent. reduction in the reversed bending fatigue limit of a Ni-Cr-Mo steel after such a treatment, but nitriding, according to Mehr, Oberg and Teres[56] has a beneficial effect. They have stated that deposits of 0·0035 in. to 0·0045 in. were plated onto a nitrided Ni-Cro-Mo steel before any significant reduction in the reversed bending fatigue limit occurred. Further increase in the deposit thickness resulted in a progressive reduction, but in general the results obtained were better than those on un-nitrided specimens similarly treated. Frith, in some unpublished work at Bristol on a Cr-Mo steel (D.T.D.306), has shown, however, that a 0·002 in. deposit on the nitrided surface of an unnotched solid test piece reduced the reversed bending fatigue limit from $\pm 46{\cdot}2$ tons/in.^{2} to $\pm 38{\cdot}4$ tons/in.^{2} The fracture began near the junction between the nitrided case and the core in both the plated and the unplated test pieces. Frith thinks that the tensile stress in the chromium plating partially reduced the compressive stress in the nitrided case.

Since Love[10] cites Barklie and Davies as having found that lead plating before nickel plating produced a decided improvement in the reversed bending fatigue properties of the plated material, it is suggested that some soft intermediate plated layer, such as copper, might also have a beneficial effect when used in conjunction with chromium.

There is little information on the effect of chromium plating on the fatigue properties of non-ferrous alloys, but tests by Wiegand[60] on Duralumin plated to a thickness of 0·0008 in. indicated that the endurance limit at 10^6 cycles was reduced by 20 per cent. as a result of this treatment.

Although chromium plating can cause considerable reduction in the reversed bending fatigue properties, when tested under normal conditions, the corrosion fatigue

TABLE IX

THE EFFECT OF CHROMIUM PLATING ON THE REVERSED BENDING FATIGUE LIMIT OF VARIOUS STEELS.

STEEL	U.T.S TONS/□"	FATIGUE LIMIT BEFORE PLATING TONS/□"	THICKNESS OF PLATING	% CHANGE IN THE FATIGUE LIMIT DUE TO PLATING TONS/□"	REFERENCES
0·4% C	35	± 15·9	0·0012"	0%	DINNER AND FELIX [39]
0·44% C	45	± 19·0	0·0014" 0·0069"	− 13% − 11%	WIEGAND AND SCHEINOST [58]
1·6% Cr	55	± 29·0	0·0012" 0·0059"	− 29% − 21%	
Cr − Ni − Mo − V	65	± 33·0	0·0016" 0·0059"	− 8% − 13%	
Cr − Mo − V	76	± 37·0	0·0008" 0·008"	− 22% − 34%	WIEGAND [57]
1·75% Ni − Cr − Mo (S.A.E. 4340)	66	± 31·0	0·001"	− 8% TO − 50% (DEPENDING ON PLATING CONDITIONS)	MEHR, OBERG AND TERES [56]
3·0% Ni − Cr (S 11)	63	± 35·5	0·0015" 0·010"	− 50% − 50%	FRITH +
4·0% Ni − Cr (S 82) CARBURISED CASE DEPTH 0·030"	71	± 43·5	0·005"	− 56%	
3·0% C − M (DTD 306) NITRIDED CASE DEPTH 0·005"	61	± 46·2	0·002"	− 17%	

+ UNPUBLISHED WORK

properties of a steel are not so drastically reduced, and may even be increased, by this process. Cazaud[51] has shown that plated mild steel test pieces possessed the same fatigue properties at 10^8 cycles in fresh water as in air.

9.2. OTHER METHODS OF COATING SURFACES

(a) Hot Dip Galvanising—(*see* Table X)

Hot Dip Galvanising is defined as:—"Zinc coating by immersion in molten zinc."

Swanger and France[61] in some fatigue tests on a series of 0·02-0·72 per cent. carbon steels treated in this manner have shown that the reversed bending limit was reduced by as much as 42 per cent. and the axial fatigue limit by 35 per cent., the amount of reduction depending upon the carbon content and the tensile strength of the material. Sopwith and Gough[62], on the other hand, have shown that the reversed bending fatigue limit of a 0·5 per cent. C steel wire treated in a similar manner was affected much less. They found an improvement of one per cent. in the "as drawn" condition and a reduction of 10 per cent. in the normalised condition. Reductions in the reversed bending fatigue limit, however, can be caused by the galvanising of poorly finished surfaces. Shelton and Swanger[63] found a 55 per cent. reduction in the reversed bending fatigue limit when comparing some galvanised 0·75 per cent. C wire with polished wire, and on testing the wire after stripping the galvanising found no appreciable increase in the fatigue limit. Wampler and Alleman[64] found similar results in tests on galvanised wire which proved to be decarburised and roughened below the coating.

Although galvanising is not an advantage under normal fatigue conditions, it has been found to be effective in increasing the corrosion fatigue resistance of steels. For example, Sopwith and Gough[62] found that after 2×10^7 cycles in a salt spray the corrosion-fatigue properties of a galvanised 0·5 per cent. C steel in the "as drawn," or "normalised," condition were not very different from the corresponding fatigue properties in air. Under the same conditions the fatigue properties of the uncoated material were reduced considerably, the endurance limit being respectively 86 per cent. and 76 per cent. below the corresponding fatigue limit in air. Harvey[65] has shown that a galvanised 0·45 per cent. C steel tested in fresh water for 2×10^7 cycles had an endurance limit some 30 per cent. below the reversed bending fatigue limit of the untreated test piece in air, compared with the 70 per cent. reduction shown by the untreated test piece under similar corrosive conditions.

(b) Sherardising—(*see* Table X)

Sherardising is defined as:—"Zinc impregnation by contact with zinc powder at high temperature."

Sopwith and Gough[62] found that for a 0·5 per cent. C steel in the "as drawn" and normalised conditions, sherardising caused a 7-10 per cent. reduction in the reversed bending fatigue limit, but when the steel was tested under corrosion fatigue conditions the process was very effective. In the "as drawn" and coated condition, after 2×10^7 cycles in a salt spray the steel possessed an endurance limit equal to the fatigue limit of the uncoated steel in air, while the normalised and coated steel had an endurance limit only 7 per cent. below the corresponding fatigue limit in air. In the uncoated condition the endurance limit was respectively 86 per cent. and 76 per cent. below the fatigue limit in air. Harvey[66], on the other hand, found that the endurance limit of coated 0·45 per cent. C steel when tested for 2×10^7 cycles in fresh water was some 30 per cent. below the reversed bending fatigue limit of uncoated steel tested in air, compared with the 70 per cent. reduction produced on testing the uncoated steel under similar conditions.

(c) "Metal Sprayed" Coatings

Metal spraying, or metallisation, is defined as:—"A process of coating with a metal by means of a spray composed of a large number of 'atomised' particles projected by compressed air from a 'pistol.' This is automatically fed with wire or stream of powder heated in an oxygen blowpipe flame, the small molten globule at the end of the wire or powder stream being continuously 'atomised.' Any metal obtainable in either of these forms can be sprayed by this process."

To obtain good adherence of the sprayed metal coating, the surface of the component must be roughened by rough threading, grooving and knurling or shot blasting. In almost all cases the fatigue limit of the coated test pieces is greatly reduced, as shown in the results in Table XI, the amount of reduction depending upon the type of surface preparation used. According to Williams[67], reversed bending fatigue tests on a 3 per cent. Ni-V steel sprayed with a 0·4 per cent. C steel showed that preparing the surface by rough threading or grooving and knurling resulted respectively in a 45 per cent. and a 68 per cent. reduction in the fatigue limit, and shot blasting produced a 26 per cent. reduction. Under reversed torsional conditions the reductions were 54 per cent., 49 per cent. and 10 per cent. on test pieces similarly treated. This trend is confirmed by Horger and Buckwalter[68] who reported a 7 per cent. reduction in the reversed bending fatigue limit of a 0·4 per cent. C-Mn steel metal sprayed after sand blasting.

(d) Phosphate Coatings

Steel components can be given a rust-proof surface by being immersed in a phosphate bath, the treated surface being considered a good base for the application of paint, enamel or lacquer.

In the "as phosphated" condition there may be a slight decrease in the fatigue properties; Wiegand[57] has reported a 4 per cent. reduction in the reversed bending endurance limit of a Cr-Mo-V steel of 76 ton/in.² U.T.S. treated to improve corrosion resistance. Sopwith and Gough[62] also found slight changes in the reversed bending limit fatigue of a 0·5 per cent. C steel phosphated and then enamelled. In air, the steel which had been treated in the "as drawn" and polished condition showed a 7 per cent. reduction, but the normalised and polished steel showed an 8 per cent. increase in the fatigue limit when similarly treated. They also reported that the treatment resulted in a considerable

improvement in the endurance limit on the basis of 2×10^7 cycles in a salt spray. Similar results were obtained on test pieces enamelled without being previously phosphate treated. This seems to indicate that the enamelling, rather than the phosphating, was responsible for the changes in the fatigue properties.

10. The Influence of Temperature

The influence of temperature changes on the fatigue properties of metals must be taken into account whenever components are exposed to such conditions. Not only must the basic material be able to withstand the change in temperature but also, any surface treatment given to increase the fatigue properties must also be unaffected. From the aero-engine metallurgist's point of view, the influence of temperature on a metal usually infers the effect of increasing, rather than decreasing, temperature. On the other hand, the airframe metallurgist may well be interested in the effect of sub-zero temperatures.

10.1. THE EFFECT OF ELEVATED TEMPERATURES

The results of axial fatigue tests on a number of normalised or tempered plain carbon steels have been collected by Gough[71]. They show that for an increase of 200°C. from atmospheric temperature there is a slight, but progressive, decline in the fatigue properties. In the temperature range 200-400°C., however, the fatigue properties begin to improve, rising to a maximum and then falling off rapidly. This rise and fall in the fatigue properties coincides with the rise and fall of the tensile strength of these steels over the same temperature range, the rise in tensile strength being accompanied by a decrease in ductility—the so-called "blue brittleness" range.

Cold-working and carburising both cease to be effective as aids to increased fatigue properties at temperatures above 200°C. because of the stress relieving action of the rising temperature. In cold-working, recovery of the distorted layer takes place, that is, relief of stress without any apparent microstructural change, followed by structural changes as the temperature increases. In carburising the compressive stresses produced by the austenite transforming to martensite on quenching the high carbon surface layer are reduced by progressive tempering.

Unlike carburising, nitriding does not depend upon quenching to produce a hardened case, and since the

TABLE X

THE EFFECT OF ZINC COATINGS ON THE FATIGUE LIMIT OF VARIOUS STEELS.

TYPE OF PLATING	TYPE OF TEST	STEEL	U.T.S. TONS/□"	FATIGUE LIMIT UNCOATED TONS/□"	% CHANGE IN FATIGUE LIMIT DUE TO PLATING	REFERENCES
HOT-DIP CALVANISING	AXIAL	0·02%C	20	± 11·6	− 13·5%	SWANGER AND FRANCE[61]
		0·45%C	36	± 11·4	− 4·0%	
		0·45%C	46	± 20·6	− 35·0%	
		0·72%C	41	± 13·0	− 2·0%	
		0·72%C	75	± 28·0	− 25·0%	
	REVERSED BENDING	0·02%C	20	± 12·1	− 4·0 %	
		0·45%C	36	± 16·1	− 25·0%	
		0·45%C	46	± 20·8	− 14·0%	
		0·72%C	41	± 15·6	− 13·0%	
		0·72%C	75	± 42·0	− 42·0%	
		0·47%C	65	± 24·5	+ 1·0 %	SOPWITH AND GOUGH[62]
		0·47%C	43	± 16·4	− 10·0%	
ELECTROPLATING		0·02%C	20	± 12·1	+ 1·8 %	SWANGER AND FRANCE[61]
		0·45%C	46	± 20·8	+ 5·5 %	
		0·72%C	41	± 15·6	0 %	
		0·72%C	75	± 42·0	+ 11·0 %	
		0·47%C	65	± 24·5	0 %	SOPWITH AND GOUGH[62]
		0·47%C	43	± 16·4	− 2·0%	
SHERARDISING		0·47%C	65	± 24·5	− 7·0%	
		0·47%C	43	± 16·4	− 10·0%	
		0·2%C	43	± 20·0	− 10·0%	FRITH +

+ UNPUBLISHED WORK

TABLE XI

THE EFFECT OF METAL SPRAYING ON THE FATIGUE LIMIT OF VARIOUS STEELS

STEEL	TYPE OF TEST	SPRAYED COAT	U.T.S. TONS/□"	FATIGUE LIMIT UNCOATED TONS/□"	% CHANGE IN FATIGUE LIMIT WHEN METAL IS SPRAYED USING DIFFERENT METHODS OF PREPARATION.				REFERENCES
					ROUGH THREADING	GROOVING AND KNURLING	SHOT BLASTING	SAND BLASTING	
Ni–V	TORSION	0·4% C STEEL	47	± 16	– 54%	– 49%	– 10%		WILLIAMS[67]
Ni–V	REVERSED BENDING	0·4% C STEEL	47	± 23	– 45%	– 68%	– 26%		
C–Mn S.A.E. 1035		0·4% C STEEL	36	± 13·5				– 7%	HORGER AND BUCKWALTER[68]
S.A.E. 1035		0·4% C STEEL	39	± 17	– 19 %		– 13%		FULLER[69]
3½ Ni STEEL		13% C STEEL	—	± 18	– 27%	– 34 %	+ 32%		STILES[70]

treatment is done at 500-600°C., depending upon the alloy used, nitriding will be virtually unaffected by temperatures below that of the treatment. Also, as the core material has been exposed to this temperature it will not be adversely affected.

Precipitation hardening alloys, both ferrous and non-ferrous, depend for their increase in hardness upon the precipitation of a phase or phases in a fine form by re-heating a supersaturated solid solution obtained by quenching from a higher temperature. The fine precipitate formed produces severe straining of the atomic lattice which causes hardening of the alloy. If the alloys are held at temperatures above the precipitation temperature, then the fine dispersed particles will tend to agglomerate into larger masses, consequently relieving the strain on the lattice. This causes softening of the alloy and a reduction in the fatigue properties. This process depends on temperature and time, being relatively slow near the precipitating temperature. When the "life" of a component is measured in thousands of hours and not years, it is possible to use these alloys at temperatures somewhat above their precipitation-hardening temperatures.

At high temperatures the possibility of failure by creep must be borne in mind when combined creep and fatigue stresses are present. In some unpublished work at Bristol on wrought Nimonic 80 material subjected to a reversed bending fatigue stress with an additional 6·0 tons/in.² static tensile load at 600-800°C., Frith considers that failure occurs by fatigue at 600°C., whereas creep is the predominant factor affecting failure at 800°C. It has also been found that the change of emphasis from fatigue to creep on wrought aluminium alloy aero-engine piston materials occurs within the 250-300°C. range.

The effect of oxidation on the fatigue properties of steels used at elevated temperatures must also be borne in mind. When the oxidation products formed are porous, then the process proceeds rapidly, causing surface roughness and pitting which leads to a reduction in the fatigue properties. Once a crack forms, oxidation proceeds into the crack and failure is accelerated. The addition of chromium and nickel, especially chromium, promotes the formation of thin adherent and protective oxide films on the surfaces of the material which inhibit further oxidation, and consequently the fatigue properties are not so greatly affected. Steels generally show improved scaling resistance in proportion to their chromium content.

Brasunas, Gow and Harder[72] have shown that to obtain maximum oxidation resistance at 870°C., 980°C. and 1,095°C. the percentage of chromium must be 19, 21 and 26 respectively. They have also shown that the presence of nickel is beneficial, an addition of nickel raising the temperature at which a particular chromium content affords maximum resistance. For this reason the plain chromium or the nickel-chromium steels, together with the nickel and cobalt base alloys, are used for applications at elevated temperatures where oxidation problems are serious.

Burling Smith[73] has stated that although the maximum scaling resistance can only be expected when no deformation is present, the application of steady tensile stresses, such as with creep, does not necessarily impair the resistance of the material to any great extent. This is only true when the "self healing" properties of the protective oxide layer are sufficient to plug any cracks which may occur in the film. In his opinion the main damaging stresses are those which alternate or fluctuate, as with material subjected to fatigue conditions. Under the maximum strain in tension the oxide film is expanded and then subjected to severe compression as the initial stress is removed, or changes direction. Such compressive stresses added to the normal compression stresses of the protective oxide film may exceed the adhesive forces between the oxide and basis metal and lead to "spalling" and exposure of the metal to renewed attack.

Both reversed bending and torsional fatigue test results show a much smaller degree of scatter at elevated temperatures than that shown at atmospheric temperature. The reason for this is far from clear but it is

suggested that the soaking period before testing helps, in part, to relieve any small differences in surface stresses present in the test pieces, thus bringing them closer to parity with each other.

10.2. THE EFFECT OF LOW TEMPERATURES

The effect of sub-zero temperatures upon the notched and unnotched fatigue properties of ferrous and non-ferrous alloys has been studied by Wishart and Lyon[74], Zambraw and Fontana[75], Gillett[76] and others[77], [78], at temperatures down to −196°C. In general their results indicate that the tensile and fatigue properties increase with decreasing temperatures, but these increases are sometimes accompanied by a gradual, or sudden, reduction in the notched-bar impact strength indicating a decline in toughness which leads to brittle failure. Ferritic steels are particularly susceptible in this respect below a certain critical temperature which depends on the composition and condition of the material. The austenitic nickel-chromium steels are not affected by low temperatures, provided that the austenite remains stable, but the austenitic manganese steels are embrittled even though the structure apparently remains unchanged. Notch impact strength of non-ferrous alloys does not appear to deteriorate on lowering the temperature, except in magnesium-base and zinc-base alloys. These alloys are comparatively notch brittle at atmospheric temperatures and become more prone to embrittlement as the temperature decreases.

At low temperatures, therefore, it is not so much fatigue failure that has to be guarded against as loss of toughness leading to brittle failure under the application of impact loads.

11. The Influence of Environment

As already stated, the condition of the surface of a material has a more pronounced effect upon its reversed bending than upon its axial fatigue properties. The effects of corrosive conditions, unless suitable means are taken to combat them, lead to pitting and roughening of the surfaces with a consequent increase in stress concentration and eventual fatigue failure. Components exposed to such conditions before being subjected to fluctuating stresses can be expected therefore to fail under loads which they would successfully withstand under normal conditions.

Even greater reductions are produced if the corrosion takes place simultaneously with the application of

TABLE XII

CORROSION FATIGUE DATA FOR VARIOUS FERROUS AND NON-FERROUS ALLOYS

TYPE OF TEST	MATERIAL	SURFACE COATING OR CONDITION	U. T. S. TONS/□"	FATIGUE LIMIT OR ENDURANCE TONS / □"			% CHANGE ON TESTING IN		NO. OF CYCLES	REFERENCES
				AIR	FRESH WATER	SALT SPRAY	FRESH WATER	SALT WATER		
REVERSED BENDING	0·5% C STEEL	POLISHED SURFACE	64	±25		±3		−88%	5×10^7	SOPWITH AND GOUGH [41]
	15% Cr STEEL		43	±29		±9		−69%		
	17% Cr 1% Ni STEEL		55	±33		±12·3		−63%		
	18% Cr 8% Ni STEEL		66	±24		±15·8		−34%		
	DURALUMIN		28	±9·2+		±3·4		−63%		
	Mg ALLOY 2·5% Al		16·5	±7+		− ✱		−		
	AL-BRONZE 9·5% Al		36	±14·3+		±10		−30%		
	AL-BRONZE 10% Al 5% Fe 5% Sn		51	±23+		±14·3		−38%		
	0·15% C STEEL		28	±15	±10·7		−29%		2×10^7	MAILANDER [41]
	0·7% C STEEL		61	±33	±10·7		−68%			
	18% Cr 8% Ni STEEL		45·5	±21·5	±21·5		0%			
	SOFT STEEL	POLISHED SURFACE	—	±13·8	±6·9		−50%		10^8	CAZAUD [41]
		0·008" Cu PLATE	—	±12·5	±12·5		0%			
		0·008" Ni PLATE	—	±10·5	±10·5		0%			
		0·008" Cr PLATE	—	±14·7	±14·7		0%			
	0·47% C STEEL (AS DRAWN)	POLISHED SURFACE	65	±24·5		±3·5		−86%	2×10^7	SOPWITH AND GOUGH [62]
		0·002" GALVANISED	65	±24·7		±23·2		−6%		
		0·0005" SHERARDISED	65	±22·8		±24·5		+7·5%		
		0·0006" Zn PLATE	65	±24·4		±21·4		−12%		
	0·3% C 3·0% Cr-Mn-Mo STEEL	POLISHED SURFACE	64	±36		±6·4		−82%	5×10^7	BARDGETT [19]
		NITRIDED 0·016"	66·5	±48		±37·2		−22·5%		

\+ ENDURANCE LIMIT ON BASIS OF 5×10^7 CYCLES

✱ NO LOAD CARRYING CAPACITY

fluctuating stresses, as shown in Table XII. Tests under such conditions are referred to as corrosion-fatigue tests and even with steels at atmospheric temperatures, no fatigue limit is obtained. The results are reported as endurance limits for a specified number of cycles, as for standard tests on non-ferrous alloys. In the normal way, speeds of testing between 1,000 and 10,000 cycles/min. do not appreciably affect the results, but since the time in the corrosive medium affects the degree of attack, the speed of testing becomes important in assessing the value of corrosion fatigue results. For example, the number of cycles to failure for a specimen tested at 1,000 cycles/min. will be considerably less than for a specimen tested at a speed of 10,000 cycles/min.

Tests by Mailänder[41] on ferritic and austenitic steels in fresh water showed that the reversed bending endurance limit of mild steel was reduced by 30 per cent., of medium carbon steel by 65 per cent. and that 18/8 nickel chrome steel was unaffected on the basis of 2×10^7 cycles. The effect of salt spray is even more dangerous than fresh water, as shown by the results obtained by Gough and Soptwith[41] on some ferrous and non-ferrous alloys. The reversed bending endurance limit of Duralumin was reduced by 60 per cent., of 18/8 nickel chrome steel by 33 per cent. and of aluminium bronze by 30 per cent. The magnesium alloy tested failed to carry any load in these tests made on the basis of 5×10^7 cycles.

The corrosion fatigue properties of steel components can be improved by using either the high chromium stainless variety, or the heat-resisting nickel-chromium alloy steels. As this is not always possible, electrodeposition of various metals can be used.

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Fatigue in Engine Design

by

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THE DIFFERENCE in outlook between metallurgist, physicist and designer on the problem of fatigue of metals is due not only to the differences in training and method but also in immediate objective, however closely their ultimate objectives may coincide.

Physicists consider fatigue in terms of crystal structure and composition of the constituents of the crystal, noting in passing that certain types of crystal structures are associated with poor fatigue strength. The metallurgist's prime interest lies in the effect of surface finish, heat treatment, the physical condition of the surface, such as degree of cold work, the effect of carburised and nitrided cases having different hardness from the core. Both think in terms of controlled condition tests, with idealised test conditions such as cylindrical test specimens, close control of changes of section, polished surfaces of only a few micro-inches surface roughness, operating in a controlled atmosphere.

Designers, by contrast, are forced to compromise both the initial conception and the final execution of the work to the dictates of available alloys, machining methods and surface finishes resulting from production operations associated with quantity manufacture, and many other limitations which reduce the capacity of the product to withstand the fatigue conditions experienced in service. The designer is assisted by, and has to decide between, the technical advice which he receives from the production shops, the metallurgist and the aerodynamic, vibration testing and stress departments, supplementing this advice from his own experience.

1. Design Aspects of Fatigue

Achievement of satisfactory life for engine components subjected to fatigue stressing has been a determining factor in aero-engine design from the beginning. The high speed, essential for light weight, subjects most engine parts to many millions of stress changes during their working lives, so design is determined by considerations affecting fatigue strength rather than ultimate failure under static loading, which is frequently irrelevant.

Herein lies one of the major differences in outlook between aircraft and engine designers. Until quite recently the strength of aircraft structures has been related to the load to produce failure by single application, correlation with the loads experienced in service being effected by the use of "load factors." With this type of loading it has not been usual to pay much attention to stress concentration effects, which can be greatly alleviated by a small amount of plastic yielding.

In engine and propeller design, on the other hand, it has long been necessary to base the stressing on the fatigue strength of the main components, making due allowance for stress concentration effects wherever these cannot be obviated by careful design and attention to surface finish.

This procedure usually provides an ample margin of strength to take care of high loading for short periods—such as gyroscopic and catapult loading cases.

Enough aircraft structural fatigue failures have now occurred, however, to make it clear that fatigue strength must be regarded as a controlling influence in aircraft design also.

The difference in outlook between the airframe structural designer and his colleague in the aero-engine field is that the former thinks in terms of "Reserve Factors" over unity and the latter hopes that despite uncertainty about vibratory loadings, the magnitude of the

actual operating stresses will leave a sufficient margin from the limiting fatigue strength of the material to prevent failure in service. Vibration is a danger to both, but this aspect of the structural problem has been more pressing in engine design.

In the design of aircraft, the size and type of machine has an important bearing on the relative importance of vibration and fatigue. In a heavy long-range bomber with an ultimate load factor of 4·0 the structure may frequently be loaded to approximately 25 per cent. of the ultimate strength of the material and even modest alternating stresses would lead to ultimate fatigue failure; in a fighter aircraft of aerobatic category, on the other hand, the high load factor of 12·0 would result in only one third of the previous value of steady stress, and a consequent reduction in the relative importance of supplementary vibratory stresses.

In contrast to this, the degree of augmentation of stress by vibration in an engine remains sensibly unaffected by the type of aircraft utilisation, and the factors which are so affected (for example, gyroscopic loading) are not usually the main ones in determining design.

The general form of the structure affects the problem since the large thin areas of sheet metal of an airframe are liable to experience fluctuating stresses from elastic instability waves which may form and lead to fatigue failure. Such problems are not often serious to the engine designer, although the rotating parts of axial jet engines, *e.g.* discs, may be subject to vibratory waves, as in the two-diametral mode of disc vibration.

2. Assessment of Conditions of Loading

Continuing the comparison of airframe and engine, although they share common types of loading, such as aerobatic manoeuvres and landing, for which stressing methods are similar, it is common practice to assess the strength of the airframe structure mainly in terms of its failing load, which is designed to be equal to the product of the working load and an arbitrary factor varying with the type of aircraft and flight case considered.

In engine design, on the other hand, the number of stress cycles to which major moving parts may be subjected is generally so high that fatigue strength is from the outset the dominant consideration in determining the design. A satisfactory jet engine turbine blade, for example, can have a higher mode of vibration (12,000 cycles/sec.) in the running range. During a 150-hour type test this blade will have withstood some two thousand million stress cycles but these are of a magnitude low enough to avoid failure.

The assessment of the magnitude of the alternating stresses involved in the higher complex modes of vibration of a turbine engine blade cannot be calculated by the routine methods available to the designer. A similar condition exists in the problems associated with flutter, where the parameters which determine satisfactory operation and freedom from fatigue are not yet elucidated. This situation is relieved, however, by the fact that it is comparatively easy to make sample blades for determining vibration characteristics experimentally.

There are, of course, some engine parts where fatigue stressing is not the principal consideration as, for example, the turbine disc, but in this instance it is usually impossible to avoid some degree of plastic flow, and stressing has to be based on "creep" considerations, which lie outside the scope of the present discussion.

Generally speaking, the engine designer's chief difficulty lies in assessing the magnitudes of exciting forces and their degree of resonance with natural frequencies of the system under consideration. For example, the modes of vibration and natural frequencies of compressor and turbine blades can be determined experimentally with considerable accuracy but this is not enough for design: one must also know the magnitudes of the excitations produced by non-uniformities in flow produced by other rows of blading, struts and so on, or at least the relative possibilities of failure from these causes. Too often this information only arrives after the engine is built, and then only after elaborate and difficult strain gauging has been done. A similar comment applies even more forcibly to the problem of aero-elastically excited vibrations, such as flutter.

The turbine engine designer has a valuable precedent in the immense amount of fruitful research work done on propeller/crankshaft vibration study. However difficult the application of this strain gauging technique to a red hot turbine blade may be, it offers at present the only source of information, apart from experience, on the magnitude and origin of the vibratory loading to which the blading is subjected.

3. Effect of Damping

In considering damping it is important to distinguish between (*i*) the effect of damping in reducing fatigue stressing by a reduction of vibration amplitude and, (*ii*) the possible relationship of damping to fatigue strength. The usefulness of high damping capacity in reducing response to vibratory forces may be limited by a reduction of fatigue strength.

In general, variations of heat treatment which improve fatigue strength tend to reduce damping capacity in most steels, but because damping coefficients vary with stress levels there is no inherent relationship between damping and fatigue strength.

A correlation is often sought between damping capacity and notch-sensitivity but there does not seem to be any essential connection between the two. Some plastics, for example, have high damping capacity but are very notch-conscious, and the same applies in a lesser degree to some light alloys.

4. Diversity of Parameters

The difficulty of providing for fatigue effects in design is greatly enhanced by the large number of interrelated parameters affecting fatigue life. The overall result is so complex that it is impracticable to encompass all combinations by laboratory fatigue testing.

The most important influences are:—

(*a*) Type of stress—direct or shear.
(*b*) Range of stress variation—fluctuating or alternating.
(*c*) Stress concentration.
(*d*) Type of material.

(*e*) Effect of surface treatment.
(*f*) Effect of corrosion—including fretting.
(*g*) Temperature.
(*h*) Effect of size.
(*k*) Effect of speed.

These will now be considered individually.

(*a*) *Type of Stress*

The fatigue strengths of polished laboratory specimens can be determined in the usual way for either direct or shear stresses, and the results can be applied to the design of those engine parts which are subjected to only one kind of stressing.

For the case of combined direct and shear stressing, a considerable literature is available to the designer; outstanding is the classic work of Gough and Pollard, summarised in Dr. Gough's Presidential Address to the Institution of Mechanical Engineers in 1949[1].

(*b*) *Range of Stress Variation*

Dr. Gough's address also deals with the more general case in which the (algebraic) mean stress is not zero, and goes on to review the additional complexities caused by the combination of bending and torsional cyclic stresses whose means are not zero.

Although this monumental research is a most valuable aid to the designer wrestling with the complex stressing usually encountered in practice, its assistance in the initial design stage should be supplemented as soon as possible by full-scale testing of actual components on special testing facilities which simulate operating conditions as closely as possible, or better still, by overload testing in the engine itself.

(*c*) *Stress Concentration Factors*

The study of stress concentration effects has probably fascinated more investigators and evoked a greater effort than any other single aspect of the strength of materials. Elegant and ingenious techniques using transparent models and polarised light have been devised to obtain factors which agree well with theory in those cases where it is possible to calculate the stress distribution, and where the shape and material of the component conform sufficiently closely to the assumption made in formulating the theory.

Fortunately for the engine designer, the high concentration factors predicted by theory and confirmed by these techniques are only approached in the harder and more brittle engineering materials. In most engineering applications the practical stress concentration factors (or more accurately, the fatigue strength reduction factor) vary widely with stress range, and are dependent on the shape of the component and the characteristics of the material. Materials such as cast iron, for example, have so many inherent weaknesses that additional stress raisers do not have much effect. In most steels, on the other hand, the practical stress concentration factor increases steadily as the stress changes through the fluctuating to the alternating range. The same is true of light alloys, although these alloys, particularly the magnesium alloys, tend to be more "notch-conscious" than most steels.

The reason for the variation of effect of stress concentration with stress range lies in the capacity of the material to redistribute fluctuating stresses by plastic flow at the more highly loaded points. Stress reversals, however, tend to cause corresponding reversals of plastic flow, which would initiate work-hardening tending to embrittle the material locally, thus eventually preventing relief by plastic deformation. If the local value of the concentrated stress exceeds the fatigue strength of the material a crack will start, resulting in a concentration factor at the sharp root of the crack so high that propagation into fields of lower stress can take place.

Hence accurate determination of the concentration factor for a particular application is, in general, extremely difficult. Fortunately, however, the practical stress concentration factor seldom exceeds three for good design, even with a fully-alternating stress range, although there are not many stress raisers in practice which have a concentration factor less than 1·5. Determination of the actual value between these limits is at present largely a matter of engineering judgment based upon experience.

(*e*) *Surface Treatment*

Surface condition has an important influence on fatigue strength, especially in regions of stress concentration. A superficially smooth surface produced by a final polishing operation, however, does not necessarily guarantee that all the damage caused by previous heavy machining operations has been removed. In other words, it is essential to know the sequence of machining and heat treatment procedures before the quality of the product can be assessed.

Surface treatments, such as shot peening, cold rolling, carburising and nitriding which result in surface hardening, tend to increase fatigue strength by introducing surface compressive stresses. The start of a fatigue crack then generally occurs at the inter-face between core and case, and for this reason the component is often rendered less notch-sensitive.

Nearly all types of surface plating, on the other hand, result in some loss of fatigue strength. In some instances this loss may be as much as 30 to 50 per cent.

(*f*) *Corrosion Effects*

The effect of corrosion in reducing fatigue strength is greatly increased when corrosion occurs while the component is stressed. Indeed, under conditions of stress corrosion there is a steady decline of fatigue strength with time.

The highly stressed parts of piston engines have been relatively immune from corrosion effects because they operate in an oily atmosphere. On the other hand corrosion is an important consideration in turbine and compressor blading.

In both types of engine certain parts, such as shafts under ball and roller bearings and toothed couplings are subject to fretting corrosion which tends to impair their fatigue strength.

(*g*) *Temperature*

The effect of temperature on fatigue strength is of growing importance, not only to the engine designer, in relation to components such as turbine blades, where it may be of dominant importance, but also to the aircraft designer now that aircraft speeds are moving into the supersonic region.

(*h*) *Effect of Size*

Since the actual components are generally much larger in cross section than the laboratory specimens from which their strength is estimated, the question of size effect is important. Experiment has shown that many materials exhibit an appreciable increase in fatigue strength as the size of the specimen is reduced, and it has also been found that stress concentration factors tend to be somewhat higher for large specimens than for small ones. The evidence is somewhat contradictory, however, and the only effective method of assessing the true fatigue strength of actual components is by full-scale testing.

A factor tending to reduce the fatigue strength of large and complicated components is the presence of adverse residual stress systems resulting from heat treatment or machining operations. In many cases stress relieving treatments are necessary to deal with this situation.

(*k*) *Speed Effect*

Up to about 1,000 cycles per second, vibration frequency has little effect, but for frequencies of 10,000/sec. or more there may be an appreciable increase in fatigue strength. Insufficient data is available as yet for advantage to be taken of this in the design of parts such as compressor and turbine blading.

5. Fatigue Life

There is a tendency in engineering discussions to differentiate between static strength and fatigue life on the assumption that fatigue life is measured in millions of stress cycles. It is therefore perhaps as well to state explicitly that the fatigue history of a component begins with the first application of stress and fatigue failures can, and do, occur after only a few hundred stress cycles.

An interesting example can be quoted from a series of fatigue failures which occurred on a case-hardened engine part subjected to very little fluctuation of stress during normal operation. It was eventually discovered that abnormal conditions during starting had initiated a crack, which thereafter spread by fatigue in the usual way. During laboratory investigations, however, it was found that the part would crack after some 250 applications of a load only two-thirds that required to crack the case by a single application, and it was especially noteworthy that an examination, only some dozen cycles before failure, did not disclose the slightest sign of damage of any kind.

The effect of under-, or over-stressing in the early fatigue history of a part may be beneficial or detrimental, according to the amount and the type of material. The importance of knowledge on these effects in relation to operations such as cold straightening, can hardly be over-emphasised. As already mentioned, however, some over-stressing can be beneficial as, for example, cold-working which produces surface compressive stresses. An excellent example is the high fatigue strength of cold-rolled threads.

Where fatigue history of a part is fairly uniform, it is possible to attempt some correlation between service life and the shape of the fatigue curve for the material as determined in the laboratory. Great caution must be observed in these comparisons, however, because many factors may completely change the shape of the curve, the effect of corrosion being perhaps one of the most important.

These considerations bring us to the greatest difficulty in allowing for fatigue in engine design—that of estimating the working life of a part subject to fatigue. For materials with a defined fatigue limit for indefinite life, it is possible to compare the estimated fatigue stress (after allowing for stress concentration) with the fatigue limit and thus arrive at a factor of safety, but in the many cases where the fatigue curve has no horizontal portion, the "scatter" in life of apparently identical parts performing the same duty may be very large.

Because of these uncertainties, confirmation of fatigue strength calculations must be obtained by actual engine tests, and since the engine cannot be run indefinitely to prove unlimited fatigue life, over-load testing must be adopted. This can be extremely expensive so that efforts are usually made to develop rigs for testing individual components. Only too often, however, these rigs have to be used to compare the merits of possible cures for fatigue failures arising on the test bench.

6. Design Maxims

Many of the points affecting the stress department have been dealt with in the foregoing and it is now proposed to enumerate a few design maxims. Most of these should be self-evident but judging by the constant recurrence of avoidable fatigue dangers, their importance does not appear to be generally appreciated. Unquestionably the foremost design maxim should be "Remember—no sharp corners." Although this rule is especially important for re-entrant or internal radii, it should be applied also to external sharp corners, despite the fact that theoretical stress values at external corners may be very low.

The removal everywhere of sharp corners by a standard amount (usually 0·010 in. radius) on engine components should be so standardised that a special drawing instruction is required if a sharp corner is permissible and functionally necessary. In support of this suggestion, a case can be cited of a crankshaft which failed in under 40 hours because an oil hole was left "as drilled" with a sharp corner, compared with the indefinite life of thousands of similar shafts having the large radius called for at this point. On every wall of every drawing office there should be a notice—"Remember the Fillets."

The second maxim is, "Err on the generous side with radii": they are seldom made too large. For

example, the fillets at the bottom of the gear teeth should be as nearly semi-circular as possible, and serrations, splines, threads and so on, should have the largest practicable radii. This may seem self-evident but the standard American thread is notoriously bad from the fatigue standpoint.

The third maxim is, "If possible avoid all sudden changes of section by introducing gradual blending." Neglect of this maxim can result in seriously weakening a part by the addition of metal intended to strengthen it. The addition of relatively thin webs or ribs to a casting is a case in point. The load is often concentrated at the edge of this rib, resulting in high fatigue stressing and early cracking. Sometimes these cracks are so effective in re-distributing stress that a drilled hole is enough to stop them spreading.

The fourth maxim relates to the importance of surface treatment and finish. Drawings should indicate clearly the standard of finish required on the highly stressed components, and the areas over which hardening or other surface treatment is required.

Fifthly, in selecting the material from which to make a component, attention should be paid to the degree of stress concentration present and the nature of the loading. Materials of very high tensile strength often have a relatively low fatigue strength and are very "notch conscious," so that materials of lower ultimate strength sometimes result in longer life.

Finally, good design backed by sound workmanship do not by themselves guarantee satisfactory behaviour in service. The best efforts of the designers, production engineers and engineering specialists must be followed by proper care in operation and maintenance.

Many fatigue failures have been caused, for example, by improperly tightened nuts.

7. Future Development

Unquestionably one of the greatest blessings in the development of aero-engines, would be the discovery of a non-destructive method of determining whether a component has been operating within a stress limit which will result in satisfactory life. There is at present no method of determining whether or not a part is going to break; it can operate for a long time under conditions which will eventually result in failure without showing any form of deterioration detectable by methods known at present.

Work being done by Gough and Wood just before the 1939-45 War on x-ray diffraction pictures of crystal structures in mild steel was promising in this respect; it was found that alteration of the crystal structure was produced by stresses exceeding the fatigue limit, while lower stresses did not produce this alteration. Unfortunately, this method was not applicable for the highly-alloyed steels used for most engine parts and no generally applicable criterion of fatigue life has yet resulted from it.

Unless, and until, some such research tool is developed to the point of practical use, development of aero-engines will continue to depend on the "make and break" technique which has resulted in the present standards of reliability and technical advancement.

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Fatigue in Aircraft Design

by

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THE PROBLEM OF FATIGUE in aircraft design is a comparatively new one so far as the airframe designer is concerned. There are three main reasons for its prominence. Firstly, the utilisation rate has increased enormously, particularly with transport types; indeed, the hours flown by a civil aeroplane in a single year nowadays far exceed those in the whole life of a bomber up to the period of the 1939-45 War. Secondly, the increased cruising speeds of modern aircraft have aggravated the loading conditions which give rise to fatigue; and thirdly, the development of modern high strength light alloys has enabled higher ultimate stresses to be used, although the fatigue resistance of the material has not kept pace with its ultimate strength.

What is the fatigue problem? An aeroplane in flight, or in landing and taxying, is subjected to fluctuating loads, usually superimposed on relatively steady loading conditions. The accumulative effect of these repeated loads can produce fatigue in the material if the tensile stresses are too high. This sounds delightfully simple, but to determine how high this tensile stress may be the following must be known:—

1. The magnitude and frequency of the loads that are damaging.
2. The true peak values of the tensile stresses, allowing for the concentrations which occur at every hole and every change of section; also allowing for stresses locked up in the material in the processes of manufacture of the finished article.
3. The fatigue properties of the material which, in turn, are influenced by the surface finish.

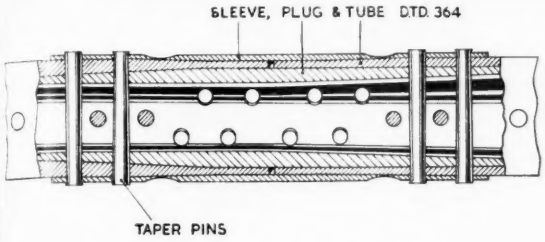

FIGURE 1. Viking outer-plane spar joint.

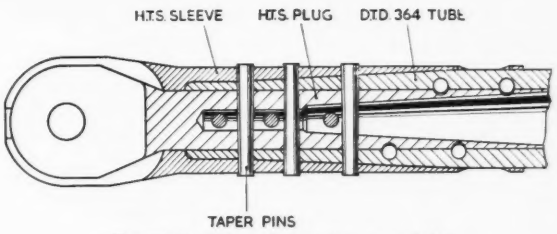

FIGURE 2. Viking inner-plane spar joint.

These are broad statements and knowledge on all three is limited. Add to these limits a fourth, relative movement of abutting surfaces which can markedly lower the fatigue life by attrition, then a fifth—a spot of corrosion—and the problem ceases to look simple, and indeed it is not.

I am going to be frank in discussing our experience with fatigue failures but first I would like to correct the suggestion in, at any rate, one section of the popular press that it is only British aircraft which are subject to fatigue and that American counterparts are mysteriously free from this trouble. I do not wish to belittle the excellence of American aircraft, but prefer the suggestion of my friend, Major Teed, that their phenomenal life is comparable with that of the axe in the Tower of London. This axe has done noble work in its time, but in its life of seven centuries it has had 14 new shafts and 24 new heads.

Our first experience with airframe fatigue failures occurred in the years 1942-1944, when some 20 Wellingtons were known to have been lost through fatigue failures of the main spars, in many cases occurring when high gravity loads were imposed. Most of these failures occurred at the same place—at the inboard end of the lower spar boom joint between the inner and outer planes. The number of flying hours before failure was, by current standards, extremely small. Examination of all the joints in the spars revealed the presence of small fatigue cracks in varying degrees and in many cases the most severely affected joint was the top break joint, *i.e.* in the booms normally regarded as compression members. This led to the thought that it was probably landing and taxying loads which initiated the fatigue cracks. Research work by J. B. Lambie at the Royal Aircraft Establishment, in which the stresses in the booms were measured in flight, confirmed this view.

The remedial action which was taken appreciably prolonged the fatigue life. The spar boom material was changed from DTD.273, which gains its high strength properties mainly by cold work, to DTD.464 which is a double heat-treated material. The damping characteristics of the undercarriage were changed and some small changes were made to detail design of the joint. Many Wellingtons are still doing useful service in the R.A.F.

Towards the end of 1944, with the end of the War in sight and thoughts turning to post-war civil aircraft for which flying lives were envisaged far in excess of those considered adequate for military aircraft, the problem of fatigue became one of extreme urgency and considerable effort was directed towards establishing some suitable standard whereby the fatigue lives of different designs could be assessed. This led eventually to what has become known as the R.A.E. Fatigue Criterion which specified the loading conditions as $1\,g \pm 7\frac{1}{2}$ per cent. of the ultimate design load in the 50 ft./sec. gust case at design cruising speed. The number of reversals of load to be arrived at under these loadings has been suggested as two million. This figure would, I think, cover the worst conditions ever likely to be encountered, *i.e.* low flying in the tropics. This criterion has been fully discussed by Dr. P. B. Walker[1]. This endurance target is very difficult to achieve at stress levels determined by static strength considerations.

For the Viking, the mainplane structure of which closely resembled the Wellington, efforts were concentrated on improving the joint design of the spars and a sleeved joint was developed which showed a marked improvement over the Wellington. Early work was done on an outer wing joint as the loads at this region brought it within the capacity of the 20 ton Avery-Schenck testing machine, which was the only one available. The R.A.E. criterion had not by then been established, so the joint was developed under some arbitrary loading conditions. Fig. 1 shows this joint.

We succeeded in maintaining a reasonably uniform distribution of load on all pins and early work made us change the pins from parallel to taper. The significance of this was not at the time fully realised and I will comment on taper pins later.

This joint was subsequently tested at the R.A.E. to their criterion, which it met. Ironically, however, we were able to dispense with it completely by employing longer booms.

Figure 2 shows the Viking inner plane joint which is similar in design but, because of the higher loads and space limitations, high tensile steel sleeves are employed instead of light alloy. This mixture of two materials with such differing elastic properties made the joint design more difficult and to avoid overloading the end pins, "elastic" lightening holes were introduced into the outer steel sleeve to make its effective Young's modulus more comparable with that of light alloy.

When the 60-ton Avery-Schenck machine became available at the Royal Aircraft Establishment a number of these joints were tested to the criterion conditions and gave rather disappointing results. This led to a decision to withdraw two aircraft with about 7,000 flying hours from service with British European Airways and to test the appropriate pieces of spar from these aircraft under the same conditions.

Meanwhile, further new joint specimens were being tested, the results of which showed considerable improvement over those obtained from the first batch of specimens. The reason for this was finally attributed to the differences in workshop technique ensuing from the fact that the first specimens were assembled and pinned on the bench; the others were assembled and pinned in a jig which, because of its rigidity, ensured that the taper pins were driven well home and became interference fits. W. A. P. Fisher of the R.A.E. has discovered the quite startling beneficial effects to be obtained by plugging all holes with the right degree of interference fit and there is little doubt that we were getting some of this benefit fortuitously, but in an uncontrolled manner.

The results of the tests on the B.E.A. spars were encouraging in that they gave remaining lives, on the average, in excess of those of the original batch of new specimens.

Because of the evidence that turbulent flying conditions are experienced more frequently under tropical conditions than in Europe, steps were taken to obtain for test spars from a long-life Viking operated by Central African Airways, and at the same time spars from short-life aircraft.

The results of all the tests available from aircraft that have been in service are shown in Fig. 3, which plots the endurance remaining against the service life.

I should explain here that each aircraft provides four specimens as the spar boom consists of a pair of booms, side by side.

A study of this Fig. 3 shows the wide scatter of results obtained from specimens, even those taken from the same aircraft.

Another fact that will be noticed is that one specimen is shown as having zero life remaining. One of the booms from the Central African Airways' aircraft was found to be completely severed, the fatigue crack starting at one of the shear bracing bolt holes. Arrangements have been made to test a further 200 specimens; the results of 34 should be available shortly.

Figure 4 shows the results of a number of tests on the Viscount outer plane spar joint. The graph covers three series of tests—some 9/16 scale model tests and two lots of full scale. The first lot of full scale tests gave disappointing results and the fit of the pins was suspect. It brought to light the importance of detail—the washer under the nut of the taper bolts was recessed to permit its use when extracting, and insufficient rigidity allowed the taper bolts to work slack under repeated loads. The second batch of specimens had this fault corrected. Before testing they were pre-loaded to 2 *g*. It will be seen that these specimens all satisfy the R.A.E. Criterion.

Figure 5 shows the design of a number of Viscount spar joints. These all have one thing in common, the use of taper bolts in shear. Whether the joint is a break joint or a constructional joint, the use of taper bolts enables it to be assembled and dismantled with little risk of damage to the component parts. We have used a taper of one in 20 and find that we are able to control the degrees of interference fit quite accurately by

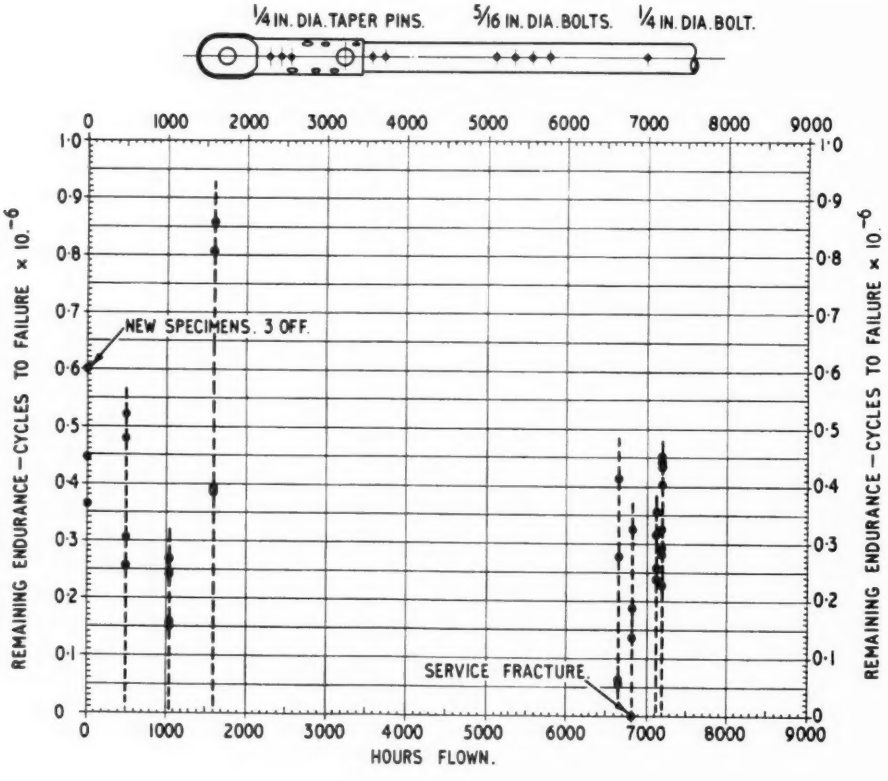

FIGURE 3. Viking centre section spar. Fatigue tests on used spars.

adjusting the nut, provided that there is a trace of lubricant on the pin. One thousandth of an inch interference is represented by a 20 thousandth axial movement, which is enough to give accurate control.

The top diagram represents the joint of the inner plane to the centre section.

The lower two joints represent alternative designs for the outer plane joint on the Type 800; the only real difference between these two is that in one case there is a fork and trident and in the other, a fork and eye.

So far only a few specimens of each of these alternative designs have been tested. The fork and eye joints have failed at $3{\cdot}3$ and $2{\cdot}7 \times 10^6$, and the fork and trident at $3{\cdot}0$, $3{\cdot}9$ and $6{\cdot}1 \times 10^6$.

I have dwelt at length on this question of joint design, but consider the standard fatigue test specimen of the metallurgist. Think of the size of the end attachments it is found necessary to use to ensure that failure shall not occur at the ends. The same freedom is not available to the aircraft designer and joint design will always be a difficult task requiring considerable skill.

When the life of the structure under laboratory conditions is known there is still the problem of translating it into a safe service life. To do this it is necessary to know how the laboratory test loads are going to compare with the average loads the structure is expected to meet in service. Work by J. Taylor [2] suggests that the number of gust loads experienced is proportional to the distance flown and is independent of speed. However, speed determines the distance flown in a given time and also, the severity of the load due to a given gust. From the fatigue damage point of view there is a "speed squared" law in the picture and the operating technique can influence the fatigue life enormously. In order to take account of varying operating techniques there is clearly an urgent need for the Fatigue Meter suggested by Taylor. The effect of altitude, which is of course one of the quantities included under operating technique, is most important. Fig. 20 of Mr. Taylor's paper suggests that a gust of 10 ft./sec. or more is met once in about 30 miles at 10,000 ft., but only once in about 2,400 miles at altitudes of 30,000 ft. or more. The evidence on which this comparison is based is admittedly very limited at present, but its significance from the fatigue point of view is considerable, as will be seen by comparing the curves in Fig. 6.

The curves show the number of gusts of 10 ft./sec., or greater, which would be met in 1,000 hours' flying by an aircraft of the Viking class cruising at 10,000 ft. and by an aircraft of the Viscount class cruising at 25,000 ft., over various stage lengths. For the purposes of the comparison the same cruising speeds in terms of E.A.S. have been assumed and the curves take account of the extra mileage covered by the Viscount class because of the higher true speed. For an average stage

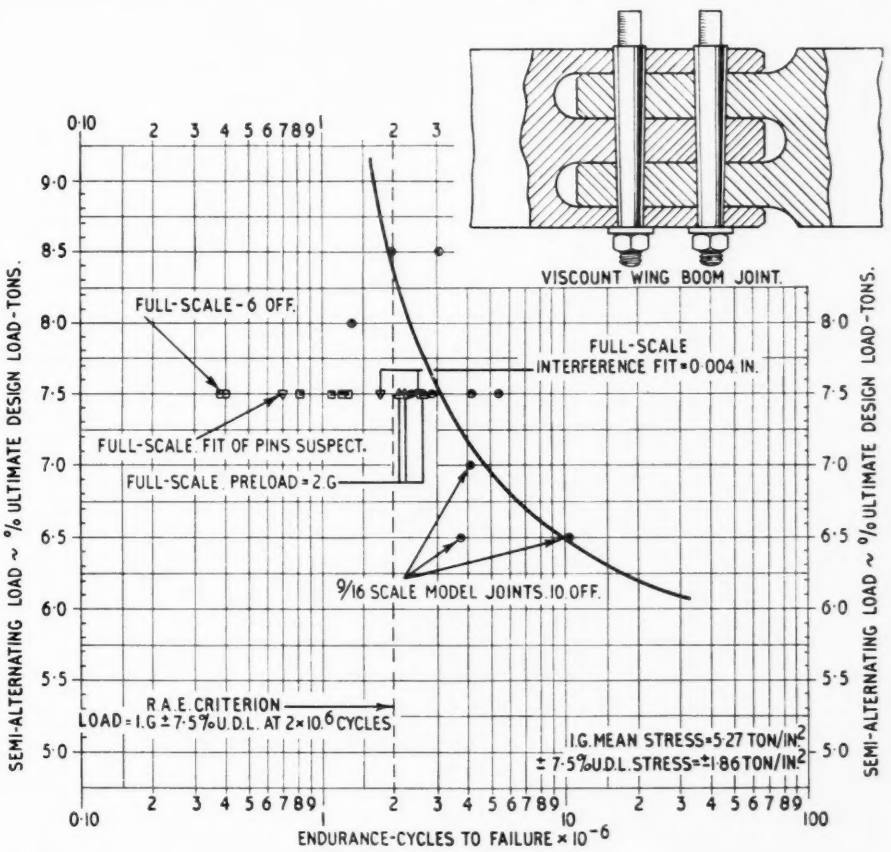
VISCOUNT WING BOOM JOINT.
FULL-SCALE - 6 OFF.
FULL-SCALE INTERFERENCE FIT = 0·004 IN.
FULL-SCALE FIT OF PINS SUSPECT.
FULL-SCALE PRELOAD = 2 G
9/16 SCALE MODEL JOINTS 10 OFF.
R.A.E. CRITERION
LOAD = 1·G ± 7·5% U.D.L. AT 2×10⁶ CYCLES
1·G MEAN STRESS = 5·27 TON/IN²
± 7·5% U.D.L. STRESS = ± 1·86 TON/IN²
SEMI-ALTERNATING LOAD ~ % ULTIMATE DESIGN LOAD-TONS.
ENDURANCE-CYCLES TO FAILURE × 10⁻⁶

FIGURE 4. Viscount main spar break joint fatigue tests. Full-scale and 9/16 scale joints.

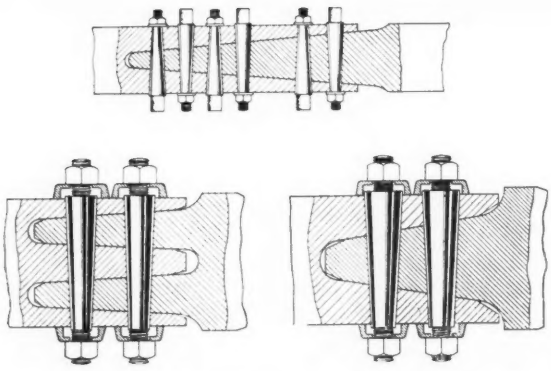

FIGURE 5. Viscount wing joints.

length of 600 miles for the Viscount class, more than 90 per cent. of the gusts are met below 10,000 ft. and more than 80 per cent. below 5,000 ft. This points the way in which the operator can do much to lengthen the fatigue life due to gust loads—firstly, by keeping low altitude flying to the minimum, and secondly, by reducing speed under turbulent conditions.

I would like to suggest some lines of approach for improving fatigue life, other than the obvious one of lowering the stress in tension members and thereby increasing the weight of structure. If aircraft could be designed so that they were free from notches, holes or stress raisers, any customer could be satisfied, at any rate on the question of fatigue life. But as this is beyond our present wit, can some residual beneficial compression stresses be induced at these points of stress concentration? I do not pretend to understand fully why the interference bolt is so beneficial; perhaps it does induce some beneficial compression stresses; it clearly reduces, if not eliminates, the onset of fatigue by attrition.

Not all loads are harmful. Indeed, the occasional high loading can do a power of good. H. H. Gardner has suggested the deliberate loading to two or more "g" in flight as a standard test flight procedure, and there is evidence that beneficial results would be obtained. However, we need to know if some other loading condition, such as that caused by a severe landing, does not undo these good results, or indeed, does not impose some harmful stress conditions. Similarly with a fin where the loading is bi-lateral, it is necessary to know the effect of occasional severe loads.

Referring again to the interference fit pin: if all holes are adequately plugged, my belief is that no harm will be done by reversal of load and I would like to see some research work done in this direction.

Another method of inducing beneficial stresses might be by shot peening, although personally I have little faith in its effectiveness in alleviating conditions where alleviation is most required, *i.e.* at stress raisers. It is possible that the more extensive use of Reduxing will help in reducing the number of stress raisers.

There are many sources of fatigue maker other than the gust case, although this has rightly received most prominence. Pressure cabins can be subjected to test

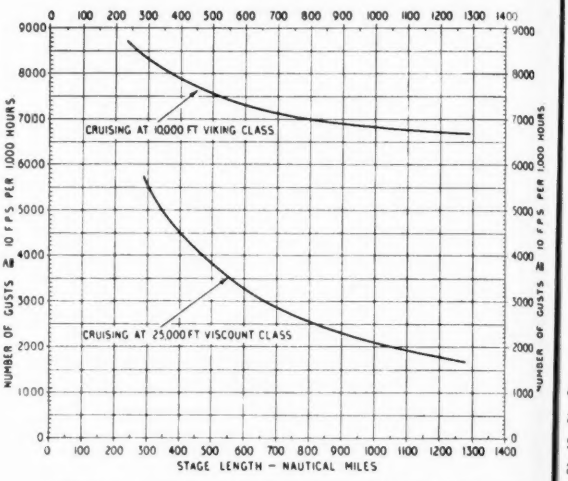
CRUISING AT 10000 FT VIKING CLASS
CRUISING AT 25000 FT VISCOUNT CLASS
NUMBER OF GUSTS Δ 10 F P S PER 1,000 HOURS
STAGE LENGTH – NAUTICAL MILES

FIGURE 6. Number of gusts encountered per 1,000 hours.

and the loading conditions are very accurately known. The plumbing of pressure systems, however, is a constant source of trouble, particularly at bends. Components which successfully withstand a static pressure of 100 lb./in.^{2} have failed in fatigue in flight at $6\frac{1}{2}$ lb./in.^{2} after a distressingly short life. This is thought to be due to very high frequency pulses from the blowers. Similar trouble is often experienced on hydraulic systems.

The jet engine has almost eliminated mechanical vibration troubles, but from the jet cone emanates high frequency pressure waves. Some tests made on one installation revealed pressure fluctuations of the order of 10 lb./ft.^{2} with frequencies of 200 to 800 c.p.s. at the fringe of the jet cone; consequently a structure in the vicinity is subjected to loadings admittedly small, but of a frequency quite outside the range of anything experienced with piston-engined aircraft—indeed, outside the range of the frequency even of our fatigue testing machines. With the advent of still more powerful engines this may prove to be a very serious problem and I would like to see some research work started on this.

Usually when fatigue failure is found, and fortunately some failures are of a minor character and, unlike the main structure, are capable of quick correction, examples of bad detail design are found.

I believe that at least half the fatigue troubles could be eliminated by better detail, but there is a deal of detail in an aeroplane.

ACKNOWLEDGMENTS

The author acknowledges the help given by the Royal Aircraft Establishment, particularly from Mr. Fisher and Mr. George. He also thanks Mr. David James, without whose help this paper would not have been given.

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Summary of Discussions

by

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1. Introduction

In preparing this summary it has seemed advantageous to depart from the sequence of discussions which actually occurred on the day. There were five discussion periods, one following each of the main papers and a general discussion at the end of the Session. Over fifty individual contributions were made to the discussions, excluding the answers given by the principal lecturers to questions addressed to them. On every occasion debate was terminated by lack of time, rather than want of subject matter. As a non-participator in the discussions, the author of this summary considered that the occasion was an unqualified success. It is hoped that the value of individual contributions will not have been lost in this attempt to summarise.

One particular contribution by R. G. Hoare, of Electro-Hydraulics Ltd., is given in full on pages 592-3.

A non-aeronautical listener might well have felt surprise that, in spite of so many expert witnesses declaring a lack of understanding of the fatigue problem, aircraft do not in fact frequently suffer from fatigue troubles. It was clear that those whose understanding was greatest were the most anxious to avoid generalisations. The atmosphere was one of cautious approach to a difficult problem.

A valuable aspect of the Session was the bringing together of designers of airframes, engines and propellers with research workers and metallurgists. The impression was formed that the engine designers and metallurgists have built up a very large background of experience which enables engines to be made with some certainty of freedom from fatigue troubles. The airframe designer is not in so satisfactory a position. Until recently fatigue has not been a major design consideration and therefore there is not the same background of information and design method to aid in the design of airframes. It is unfortunate that there seems to be no agreed explanation of the mechanism of fatigue, so that at present design rules must follow empirical approaches. This slows down the rate at which sound practice can be established. The meeting served a useful purpose in underlining major gaps in design information.

2. Crack Propagation

There was considerable discussion, but no positive conclusions, on the subject of crack initiation and propagation. General experience was that cracks started in a zone of high stress and lay in the planes of maximum direct or maximum shear stress. Although cracks frequently started at, or very near, the surface (*e.g.* at a region of high stress associated with a surface imperfection), cracks which had started internally had been observed. In certain cases flaws had been observed which had the nature of a fatigue crack in compression.

Cracks had been observed to start at as little as 20 per cent. of the total time to failure (*i.e.* the crack grew during a period of 80 per cent. of the life). This phenomenon of slow crack propagation appeared to occur when the average fluctuating stresses over the specimen were low, but with an area of very high stress concentration.

3. Practical Design Matters

3.1. FILLETS, ETC.

The strict avoidance of sharp corners was repeatedly emphasised. The value of continuous reminders to Drawing Office staff to remember the fillets was endorsed by many speakers. The particular problem of the housings for ball bearing races, the outer portion of which often has a sharp corner, was mentioned. It was suggested that aircraft and engine designers should seek to persuade ball bearing manufacturers to provide more adequate radii.

It was mentioned that the fillet was sometimes required by drawings, but missed by the workshop. Very careful inspection was necessary. Close liaison between Drawing Office and workshops was emphasised as having mutually beneficial results.

The need was stressed to avoid marking components with punch tools in positions where fatigue was likely.

A warning note was given that if a radius were made at the end of a hole it might result in the crack starting farther down the hole. The advantages of rolling or burnishing inside the hole were mentioned.

3.2. INCLUSIONS, LOCKED-UP STRESSES, ETC.

Information was sought, but was not forthcoming, on the reduction of strength in light alloy forgings due to "regions of low cohesion" (*i.e.* the regions where there is apparently no inclusion, but nevertheless the metallic bond is weak).

Reference was made to work on the effect of inclusions, some of which were found not to be detrimental.

In light alloy forgings the high "locked-up" initial stresses were believed sometimes to be responsible for poor fatigue properties. Annealing was advantageous in this connection.

In cases where the previous treatment had resulted in a surface layer of material being in a state of compressive stress, harm could be done by machining away the outer layers.

Reference was made to the use of shot-peening as a means of inducing beneficial compressive stresses. In materials where shot-peening had not so far proved beneficial, it was suggested that a gentler treatment (using low density shot) might be tried.

3.3. FATIGUE OF WELDED JOINTS

There appeared to be little evidence, either in the airframe or engine field, of fatigue strength of welded joints. Cases were quoted where welding had been abandoned on the score of poor fatigue resistance, although a particular case of steel chains was mentioned as an example of satisfactory welding.

The danger of removing the weld material (with the intention of "cleaning-up" the appearance of the joint) was mentioned. Cracks starting in the uppermost layer of weld material might be halted when they reached the next layer of weld with possible differing mechanical properties. Cleaning up the weld had been found to destroy this safety feature.

3.4. TURBINE BLADE FATIGUE

The oxidation of the surface of the blades of a turbine was considered to be potentially capable of reducing the fatigue strength, the effect depending on the blade material and the fuel used. It appeared, however, that with materials and fuels used in present turbine engines, there was no reason to expect deleterious results.

4. Possible Methods of Minimising Fatigue Danger

4.1. REDUCTION OF LOADS CAUSING FATIGUE

In certain cases fatigue could be avoided by elimination of, or reduction of, the fluctuating loads. This had frequently been the practice in engine and propeller-induced vibrations. Although such methods would not always be applicable to airframe structural fatigue, it should not be overlooked as it was generally a more economical solution than the reduction of stresses by the addition of material. Hope was expressed that turbine blade fatigue could be tackled by suppression of the loads by careful design. The practice of prohibiting the use of certain r.p.m. ranges was viewed as helpful on occasions, provided that it did not interfere with normal operational practices.

4.2. USE OF REDUNDANT STRUCTURES

Frequent reference was made to the wide dispersion of fatigue results. In structural components, at the stress levels currently employed for testing, the maximum and minimum lives measured on test were not infrequently in the ratio 10 to one. Among other difficulties this raised a serious economic problem, as the permitted "safe" life has necessarily to be a small fraction of the mean life.

It was pointed out that United States design generally tended towards redundant "inspectable" structures in contrast with British single spar designs. With the former it was safe to allow the aircraft to fly until fatigue cracks started, as the initial cracks were not catastrophic and could be seen during inspection. Doubts were expressed by some as to the wisdom of such a practice, but it appeared to be generally agreed that a move in the direction of American practice would have advantages.

4.3. NON-DESTRUCTIVE INSPECTION FOR FATIGUE

A plea was made for development of a non-destructive method by which the "percentage life used" could be determined. One of the difficulties in a complex structure or mechanism was to know where to examine, the matter being rather like searching for a needle in a haystack without disturbing the stack, and with little knowledge of the appearance of the needle.

4.4. RESTORATION OF FATIGUE MATERIAL

There was little evidence on this, but it was suggested that it would be worth while to find whether heat treatment (*e.g.* annealing) would extend the life of an already "fatigued" material. However, research on single crystals had suggested that there would be little to be gained and if cracks had already formed it was considered very unlikely, if not impossible, to restore the properties.

Giving material which had been subjected to repeated loading a period in which to rest was not thought to influence the life measurably. Some held the view that if a material were initially subjected to repeated stresses at levels below the endurance limit (*i.e.* understressing), an improvement in fatigue life would then result. Others considered that such a generalisation could not be made.

5. Test Conditions

5.1. THE GUST SPECTRUM

Several speakers emphasised the need for information about gust magnitude and frequency over the range of operational heights and typical routes. Attention was drawn to the penalty associated with the assumption of a single world-wide atmospheric condition, when it seemed possible that route or altitude might result in a 4 to one difference. The need was mentioned to take due account of the response of the aeroplane (including its flexibility) on the loads induced by a given gust.

A speaker suggested that instead of the present test conditions used for airframe components (namely a fluctuating stress of $\pm 7\frac{1}{2}$ per cent. of the ultimate design load corresponding to the 50 ft./sec. gust) it would be preferable to test to loads corresponding to up and down gusts of 10 ft./sec.

5.2. CUMULATIVE DAMAGE ASSUMPTION

Criticisms were expressed on the validity of the cumulative damage assumption, but no alternative rule had emerged. The view was expressed that, within the

limits of the load spectrum applied to aircraft wings, the present simple assumption was at least "better than nothing."

5.3. RATE OF LOADING

It was believed, from fairly scanty evidence, that if the fluctuating loads were applied at a frequency between 500 and 3,000 cycles/min., the fatigue strength would not vary with rate of loading. At a frequency of 10,000 cycles/min. a small increase of fatigue strength had been observed. Doubts were felt whether at higher rates still the fatigue strength would deteriorate or not. At lower rates, *e.g.* 100 cycles/min., reduction of strength might result and external conditions, such as temperature, would have more influence.

5.4. FACTORS OF SAFETY

Reference was made to the difference between airframe and engine practice on factors of safety. In the airframe case the safe life was usually set at a value between 40 per cent. and 50 per cent. of the mean life, and this was believed to be equivalent to applying a factor of safety on stress of about 1·25. Engine practice varied according to circumstances, but in so far as a general figure could be quoted, a factor of 2 on stress was considered reasonable. This factor was over and above any stress concentration factor.

It was pointed out that if the operating stresses were high, and a large factor was then applied to the test loads, there was a likelihood of plastic distortion round points of stress concentration. This would cause a redistribution of stress and the test conditions would no longer represent a proportional increase of operating conditions.

Under high stress conditions (on the steeply sloping part of the *S-n* curve) the scatter of test results, whether stated in terms of load or life, was of a reasonably measurable magnitude. Under low stress conditions (on the flat portion of the *S-n* curve) the scatter measured in terms of life became extremely large, but in terms of load was still a reasonable quantity. Because of this, it was suggested that a factor on load would be preferable to a factor on life.

In airframe components which were not specifically tested for fatigue, but which in service received repeated loads, *e.g.* landing gear retraction mechanism, it was suggested that the normal factor of safety of 1·5 on the static limit loading might prove an inadequate safeguard against failure.

6. Test Methods

6.1. TESTING MACHINES

Instances were cited where nominally identical specimens had been tested in machines of different types and different fatigue results had been obtained. It was suggested that scatter between test machines was as great as scatter between test specimens. The test machine scatter appeared to be associated with differences in calibration of the machines, differences in setting up the specimens and possibly differences between operators' technique. It was evident that much work was in hand to compare machines.

6.2. USE OF SCALE MODELS

A strong plea was made for research on the use of scale models. This was particularly necessary when the aircraft structural parts were large and expensive. Some evidence was quoted of satisfactory use of half-scale models of airframe components but apparently there has been little use of models in engine fatigue exploration.

6.3. ACCELERATED FATIGUE TESTS

The question was raised of speeding up fatigue tests by methods other than increasing the frequency of loading. Low temperature testing was not considered to be effective for this. Reference was made to the Renslaer method which depends on the change of coefficient of expansion of metals associated with repeated loading. This method was limited in practice by the extreme accuracy required in measurement.

7. Helicopter Problems

Little time was devoted to the special problem of the helicopter rotor and transmission. In certain respects the helicopter problem was analogous to the engine, in that the loading conditions were capable of being assessed with some precision. However, the existing state of affairs was such that permitted lives of helicopter components were exceedingly short. Blade failure was likely to have catastrophic effects so no liberties could be taken in establishing life. In order best to overcome the conflict between safety on the one hand and uneconomically short life on the other, the testing of several components was necessary. Because of the problem and cost of making numerous full-scale tests, model testing was proposed as a possibility.

8. Dissemination of Information

It was evident from the discussions that a considerable volume of *ad hoc* testing was being undertaken by the Industry and other organisations. A plea was made that a great effort should be made to publish the data so obtained.

The Society's work was described. With the assistance of a representative committee, the Society's staff was hoping to publish design information in convenient forms. To this end, the Society was seeking the maximum assistance from firms and research organisations to make available the results of general *ad hoc* testing. All this material would be examined and, so far as possible, correlated. It was then hoped to publish that which would be of general value.

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LIGHT ALLOY JACK TESTS

The use of light alloy for hydraulic jacks, particularly for thin walled cylinders, has led to trouble in many cases because of the short life experienced under working conditions. It must be remembered that hydraulic components, unlike most others on the aircraft, meet their full design loads very frequently. An extensive programme of fatigue testing of jacks has revealed that, while some failures are due to remediable design defects, there is an inherent quality of the light alloy which makes it necessary to use it with extreme caution in this application.

In an endeavour to assess the effect of this peculiarity of light alloy and to determine the most suitable form of light alloy for jack design, a series of tests is being made under contract to the Ministry of Supply, with whose permission the following details are given.

Three firms have provided between them eight different forms of light alloy bar 3 in. in diameter, from which have been made the specimen cylinders shown in Fig. 1. Half of the cylinders have been scratched longitudinally on the bore in four places to a depth of 0·008 in. Two cylinders of each batch have been subjected to a static pressure to failure. The other cylinders have been subjected to repeated applications of pressures varying from zero to predetermined values until failure has occurred.

Figure 2 shows results obtained with unscratched cylinders made of four of the materials, which were all obtained from the same supplier. If, for comparison, it were assumed that the cylinders were being used in a 3,000 lb./in.^{2} hydraulic system, the comparative results shown in Fig. 3 were obtained.

The introduction of the longitudinal scratch, in order to form a stress concentration representative of an internally scored cylinder, had led to some rather confusing results. On one material, five of the seven scratched specimens failed away from the scratch.

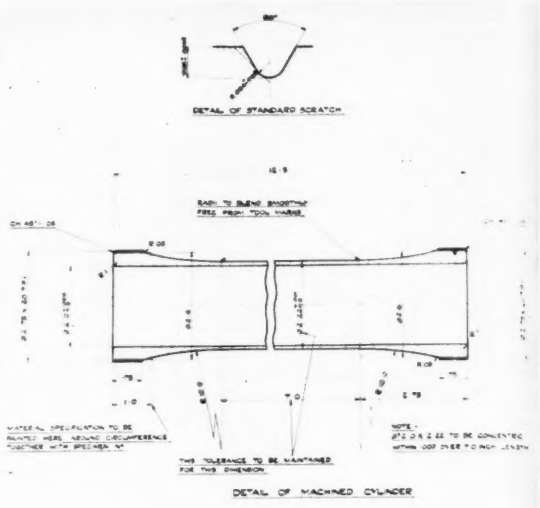

FIGURE 1.

The scatter of results was fairly wide and seemed to indicate, from the results shown, that a factor of 3 should be applied to the life obtained from test to give a safe operating life. The test results on specimens with stress concentrations, showed an even wider scatter and might need a factor of as much as 10 to cover the possible variation between test life and operating life.

Material	*R.F.*	*Minimum Life*
R.R.56 (*Forged*)	1·12	$4{\cdot}3 \times 10^4$
D.T.D.364 (*Extruded*)	1·18	13×10^4
D.T.D.364 (*Forged*)	1·24	$2{\cdot}6 \times 10^4$
D.T.D.683 (*Forged*)	1·35	$3{\cdot}3 \times 10^4$

FIGURE 3.

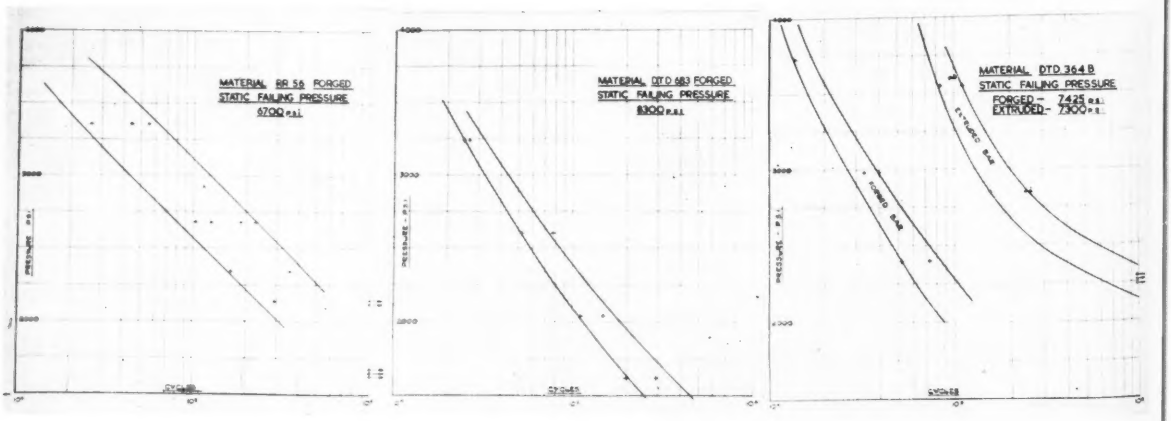

FIGURE 2.

FIGURE 4.

From the results obtained so far, it appeared that the extruded material was showing properties superior to those of the forged material. If this were borne out by the rest of the programme, it might mean that they would be able to use light alloy for jack cylinders, only where they could use extruded bar with separate end fittings or backward extrusions, which would include one end fitting. Referring to the forged materials only, it appeared that the use of the higher strength materials to obtain an improved reserve factor might have an adverse effect upon the life of the components.

These comments were based on a few results obtained at an early stage of this programme and much more work must be done before the conclusions were confirmed. When that stage was reached, they would then have an indication of the next step to take towards designing a reliable light alloy jack.

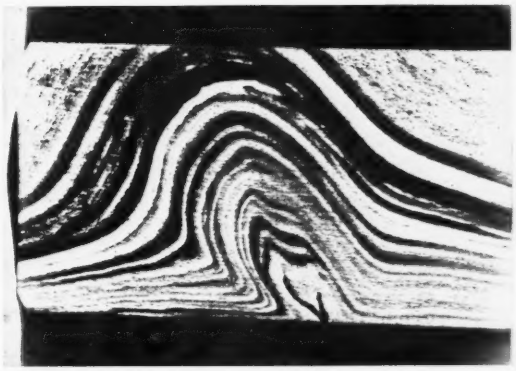

FIGURE 5.

FORGING TECHNIQUE

An interesting type of failure occurred recently in an undercarriage member which was made from a light alloy forging. Despite the attention paid to surface finish, maintenance of reasonable design stresses and avoidance of any conditions which might be expected to cause fatigue failures, after two or three years service cracks appeared in the surface of the forging and always roughly at the same position. On cutting up the forging and polishing and etching the section, they found the picture shown in Fig. 4. It appeared that the forgers, in making their use to go into the dies, had hammered the end of the bar and caused a fold which had not been worked out during subsequent forging. An enlargement of this defect (Fig. 5) showed how the crack had started at this change of grain flow.

TECHNICAL NOTES

This section of THE JOURNAL *is intended for short papers, comments upon published or unpublished papers with a limited circulation, interim results of research in hand or notes for further research, or comments on notes already published, or letters raising technical points or asking questions. Contributions, which will be eligible for Journal Premium Awards, will normally be published within two months of being received.*

The Flexural Centre or Centre of Shear

by

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THIS paper reviews work on the flexural centre of elastic cantilever beams and contains a number of hitherto unpublished results, including a formula giving the position of the flexural centre in terms of the Prandtl torsional stress function.

1. INTRODUCTION

The appearance of the note[1] by Jacobs has prompted the preparation of a review of this subject which is all the more desirable as several of the investigations made shortly before the 1939-45 War were never published and others, although published, seem to be in danger of being overlooked. The present paper contains a number of hitherto unpublished results. The question of nomenclature is worthy of mention since general agreement is lacking. Some of the names used are *flexural centre*, *centre of flexure*, *elastic centrum* and *centre of shear* but a complete search of the literature would probably lead to the discovery of yet other names. The terms "centre of shear" and "elastic centrum" seem vague and fail to indicate that the point referred to has any special relation to flexure. We shall here use the name "flexural centre" exclusively.

2. GENERAL DISCUSSION OF THE FLEXURAL CENTRE

Consider an untapered cantilever beam with a suitably anchored root R and loaded only at the tip T by surface tractions whose resultant is a force W perpendicular to the axis of the beam. Take a plane normal section of the beam at a point S which may coincide with T. The displacement at S will have a component perpendicular to the section and varying over it, but we need not consider this further here. The displacement at S perpendicular to the axis may be analysed into components as follows:—

(*i*) A rigid body displacement without rotation. This will be called the *translation* of the section.

(*ii*) A rigid body rotation about a chosen point B of the section. This will be called the *twist* of the section.

(*iii*) A distortional displacement, in general having two components which are *defined* to be zero at B.

These definitions leave the magnitude of the twist undetermined but we shall now take the principal axes of strain at B in the section and define the twist as the rotation of these axes about an axis perpendicular to the section. Since the principal axes remain at right angles in the state of strain this defines the twist uniquely. However, the magnitude of the twist will, in general, depend on the choice of the point B. The magnitude and direction of the translation of the section will also depend on this choice. Evidently the distortional displacements are now measured relative to a rigid plane which may be supposed pinned to the section at B and which rotates with the principal axes of strain at B.

Now let S coincide with T. We define the flexural centre F as such that the twist at T is zero provided that the line of action of W passes through F. Clearly the position of F will depend on the choice of B and there must, in strictness, be some restriction on the distribution of the surface tractions whose resultant is W and on the fixing conditions at the root. As explained later, we shall take B to lie at the centroid of the section.

3. THE SAINT-VENANT THEORY OF FLEXURE

So far as the writer is aware, all theoretical discussions of the flexural centre have been based on the theory of the flexure of uniform cantilever beams of isotropic material given by Saint-Venant[2]; an account of this theory will be found in Love's treatise[3]. It is therefore necessary to consider the nature and status of this theory.

In the first place, the theory is *general* in so far as the sectional form of the cylindrical cantilever considered is arbitrary. On the other hand, the theory is *restricted* in the following respects:—

(*a*) In accordance with the usual assumptions of the theory of elasticity, second order quantities in the strains are neglected.

(*b*) The beam is loaded at the tip only.

(*c*) The normal stress or "fibre stress" Z_z is as given by "the engineer's theory of bending," while the stresses X_x, X_y and Y_y are all zero. Here the axis OZ is parallel to the axis of the beam while OX and OY are perpendicular axes lying in a normal section of the beam.

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(*d*) The shearing stresses X_z and Y_z are independent of the spanwise co-ordinate z (but they are unrestricted functions of x and y to be determined by the equations of the theory of elasticity).

These assumptions imply that the load at the tip is applied by surface tractions which are distributed over the end of the beam in a special manner (in accordance with (*d*)). They also imply that the support at the root is of such a nature that (*c*) and (*d*) can be satisfied.

As a matter of fact, the load at the tip of a cantilever is never in practice applied exactly as required by the theory, nor do the root conditions conform with the theoretical requirements. It might then be supposed that the Saint-Venant theory had no practical validity or usefulness, but this is not so. Provided that the beam is not short, *i.e.* provided that the overhang is much larger than the greatest linear dimension of the section, it can be shown that the shearing stresses must be independent of z and distributed in accordance with the Saint-Venant theory, except in the vicinities of the root and tip. The theory is thus of value for slender beams (or beams of high aspect ratio) but of little or no value for stumpy beams.

It can be shown on the basis of the Saint-Venant theory that when the reference point B in the section (*see* Section 2) coincides with the centroid, so that there is no twist there when the resultant load passes through the corresponding flexural centre, the *mean* twist taken over the entire section is zero. This appears to be a sufficient reason for adopting the centroid as reference point. In what follows we shall take the flexural centre to be such that, when the line of action of the resultant load passes through it, the twist at the centroid is zero, it being understood that the whole investigation is based on the Saint-Venant theory of the flexure of cantilevers. A point called the *centre of least strain* is also mentioned later. This is such that when the line of action of the resultant load passes through the centre the displacements are such that the aggregate work which would be done by torsional surface tractions in these displacements is zero, where the torsional surface tractions on the end of the beam are distributed in accordance with the Saint-Venant theory of torsion. It can be shown that the flexural centre and centre of least strain would coincide if Poisson's ratio for the material of the beam were zero.

4. SYMMETRICAL SECTIONS

Suppose that the section of the cantilever has an axis of symmetry AA' and that there is no lack of symmetry in the supports at the root. Evidently any system of surface tractions applied at the tip section and distributed symmetrically about AA' will give no twist at the centroid O, which must lie on AA'. Hence the flexural centre must lie on the axis of symmetry. If the section has more than one axis of symmetry the flexural centre must be the point of concurrence of such axes and this is the centroid. For example, this is true when the section is an ellipse (two axes of symmetry) and when it is an equilateral triangle (three axes of symmetry). Such cases are trivial from the present point of view, but the case of a single axis of symmetry is important.

5. THEORETICAL INVESTIGATIONS OF THE FLEXURAL CENTRE

The earliest investigation of the flexural centre known to the writer is that of Griffith and Taylor[4]. These authors considered a thin symmetrical section and based their work to some extent on physical intuitions suggested by the soap-film analogy of a two-dimensional potential problem. They concluded that the flexural centre lay upon the axis of symmetry at a distance $\bar{\bar{x}}$ from the centroid given by

$$\bar{\bar{x}} = \frac{\int xt^3 dx}{\int t^3 dx} \quad . \quad . \quad . \quad (1)$$

where t is the ordinate of the upper boundary of the section measured from the axis of symmetry OX (so t is half the thickness at the abscissa x) and the origin O is the centroid. The formula implies that the flexural centre coincides with the centroid of the cubed ordinates*. McKinnon Wood and Perring[5] supported the formula (1) by an argument in which they supposed the beam to be cut into thin slices by planes perpendicular to OX and parallel to the axis of the beam. They argued that if the slices were loaded in proportion to the cubes of their depths they would all have the same deflection, so the beam would be untwisted. This argument has, however, no bearing on the problem of the flexural centre of the intact beam unless it can be shown that the shearing stresses X_z, which are abolished by the slicing, are of no importance and this is true only when Poisson's ratio is zero. Later investigations have shown that the point determined by (1) is not the flexural centre but the centre of least strain.

The abscissa of the flexural centre of an arbitrary solid section having OX as axis of symmetry and O as centroid was given by Duncan[6] in the form

$$\bar{\bar{x}} = \frac{\int xt^3 dx - 3\iint \Omega dx\, dy}{\int t^3 dx} \quad . \quad . \quad (2)$$

This formula is *exact*. The shearing stress function Ω vanishes on the boundary of the section and satisfies the equation

$$\nabla^2\Omega + \tfrac{1}{2}\frac{dt^2}{dx} - \frac{\eta x}{(1+\eta)} = 0 \quad . \quad . \quad (3)$$

everywhere within the section. Here t is, as before, the ordinate of the upper boundary of the section and therefore a known function of x, while η is Poisson's ratio. Duncan[6] also showed that for very thin sections (2) becomes

$$\bar{\bar{x}} = \frac{(1+3\eta)\int xt^3 dx}{(1+\eta)\int t^3 dx} \quad . \quad . \quad . \quad (4)$$

*This particular formula is unchanged when t is taken as the total thickness and the origin is any point on the axis of symmetry. The exact definitions of t and of the origin are important in later formulae.

where the error is proportional to the square of the thickness chord ratio when this is small. It will be seen that (4) agrees with (1) only when η is zero. It may be well to emphasise that x *must* be measured from the centroid in the application of (2) and (4). Duncan also applied his "thickness-parameter method" to obtain explicit expressions for approximations to the stress function Ω, containing x, y, t and its successive derivatives with respect to x, correct to the $(2n+1)^{th}$ power of the thickness parameter.

H. R. Fisher[7] showed that the position of the flexural centre could be obtained from a knowledge of the solution of the Saint-Venant *torsion* problem for the beam (*see* later). He also advocated the adoption of the centre of least strain as the authentic flexural centre and showed that the two centres coincide when Poisson's ratio is zero. D. Williams[8] adopted Fisher's ideas in a paper concerned with the bending of thin-walled tubes and channels.

Stevenson[9] investigated the flexural centre, as defined in the present paper, employing a set of stress functions devised by himself. He gave exact solutions for a number of sections, including a circular sector. When the angle at the apex is very small the sector is indistinguishable from a very slender isosceles triangle. Stevenson compared the position of the flexural centre for such a sector with that for a very slender isosceles triangle, as given by Duncan's formula (4) and concluded (erroneously) that they differed. In a later note[10] Stevenson rectified his error and confirmed that his own result, in fact, agreed exactly with that of Duncan. Stevenson[11] discussed the definition of the flexural centre and the ideas of Fisher[7]; he concluded that the flexural centre should be defined in the manner adopted in the present paper, *i.e.* as being such that a load whose line of action passes through the centre causes no twist at the axis through the centroids of the normal sections.

Duncan[12] obtained the following exact formula for the position of the flexural centre of a beam of symmetric section in terms of the Prandtl torsional stress function Ψ:—

$$\bar{\bar{x}} = \frac{\int xt^3\,dx + \frac{3}{4}\iint t^2\frac{\partial\Psi}{\partial x}\,dx\,dy + \frac{3\eta}{2(1+\eta)}\iint \Psi x\,dx\,dy}{\int t^3\,dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The function Ψ is defined by the properties

(*a*) It vanishes on the boundary of the section.

(*b*) Everywhere within the section it satisfies the equation

$$\nabla^2\Psi + 2 = 0\,. \quad . \quad . \quad (6)$$

Equation (5) can easily be deduced from (2). Since Ω and Ψ both vanish on the boundary, we have by Green's reciprocal theorem that

$$\iint \Omega\nabla^2\Psi\,dx\,dy = \iint \Psi\nabla^2\Omega\,dx\,dy$$

and in view of equations (3) and (6) this becomes

$$\iint \Omega\,dx\,dy = \tfrac{1}{2}\iint \Psi\left(\tfrac{1}{2}\frac{dt^2}{dx} - \frac{\eta x}{1+\eta}\right)dx\,dy. \quad (7)$$

Hence (2) becomes

$$\bar{\bar{x}} = \frac{\int xt^3\,dx + \frac{3}{2}\iint \Psi\left(\frac{\eta x}{1+\eta} - \frac{1}{2}\frac{dt^2}{dx}\right)dx\,dy}{\int t^3\,dx} \quad (8)$$

But, since Ψ vanishes on the boundary, we have by Green's Theorem

$$\iint \frac{\partial}{\partial x}(\Psi t^2)\,dx\,dy = 0 \quad . \quad . \quad (9)$$

and (8) can accordingly be reduced to (5). In all these formulae, it is to be remembered, t is a given function of x alone.

For a very slender symmetric section we have[13] the approximation

$$\Psi = t^2 - y^2 \quad . \quad . \quad . \quad (10)$$

where the error is proportional to the fourth power of the thickness parameter when this is small. Hence the second term in the numerator of (5) is negligible in comparison with the other two, while

$$\iint \Psi x\,dx\,dy = \iint (t^2 - y^2)\,x\,dx\,dy$$

$$= \frac{4}{3}\int xt^3\,dx\,. \quad . \quad (11)$$

Finally (5) reduces to (4), which is valid for very slender symmetric sections.

The writer is not aware of any general investigation of the influence of camber on the abscissa of the flexural centre. It is clear that the effect is proportional to the square of the camber when this is small and for ordinary aerofoil sections the effect is almost certainly negligible.

6. EXPERIMENTAL INVESTIGATIONS OF THE FLEXURAL CENTRE

An experimental investigation of the flexural centres of two steel beams of slender symmetric section was made by Duncan, Ellis and Scruton[14]. The first was of *symmetric aerofoil section* (a cubic oval) and with the dimensions

Length of overhang	36·6 in.
Chord	3·0 in.
Maximum thickness	0·231 in.
Thickness/chord ratio	0·077.

Loads were applied through a small frame clamped to the beam at the tip and deflections were measured at various distances from the root by micrometer. A small frame with two contact points, one near the leading edge and the other near the trailing edge of the section, was clamped to the cylinder and the deflections under load at the two contact points measured so that the twist at the section could be determined. This method is not free from theoretical objection as the twist is not measured at the centroid of the section. However, it is only at the tip of the beam that it is possible to observe and measure directly the twist at the centroid. The root of the beam was integral with the overhang and of

rectangular section. This was very securely clamped between large steel blocks bolted to a massive bedplate. Measurements of twist were made for various chordwise situations of the load at the tip and at various distances from the root. Clear evidence of the influence of root constraint was found, but the flexural centre of the part of the beam lying between sections distant 6 in. and 28 in. from the root agreed with that given by equation (2) to within a fraction of one per cent. of the chord. Near the base the flexural centre was found to lie nearer to the centroid than for the outer part. The flexural centre was found to be very nearly coincident with the experimentally determined centre of twist.

The second beam was a prism with a section in the form of a slender isosceles triangle and the dimensions were

Length of overhang	36·6 in.
Chord	3·02 in.
Maximum thickness (base)	0·248 in.
Thickness/chord ratio	0·082.

In general, the results were similar to those obtained with the cylinder of aerofoil section. For the part of the prism lying between 6 in. and 28 in. from the root the flexural centre was found to lie at a distance from the base of the triangular section equal to 0·143 of the chord, and this agrees with the result obtained by applying the formula (4). It may be noted that the centroid is at the fraction 0·333 of the chord from the base, while the centroid of the cubed ordinates lies at 0·2 chord from the base. For the part of the prism between the root and a section 3 in. therefrom the flexural centre was found to lie at 0·174 chord from the base, *i.e.* at a distance of 0·031 chord nearer the centroid than for the outer part of the prism. The centre of twist was found to lie a little nearer the base than the flexural centre.

For both beams the value of Poisson's ratio was determined to be 0·27 and this value has been used in applying equations (2) and (4).

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Lifing of Aircraft Equipment

by

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READING the paper by Tye and Cundall in the June issue of the Journal, I was started on a train of thought on the same lines.

I agree with the paper but feel that the argument has not been taken far enough. The main conclusion is:—

So long as a random failure is no more expensive and troublesome than a controlled replacement, it is always cheaper to wait until an item fails before replacing it. This is quite independent of the law of failure with time.

A further conclusion can be deduced:—

If a random failure is much more expensive than controlled replacement, then it pays to replace an item at some fraction of the longest expected life. (If the failure law is completely random, however, this is not true).

This is based on some calculations which are not given in detail and which form the basis of Tye's Fig. 5.

I had some doubts as to the validity of the underlying arguments based as they are on the behaviour of a specified set of components which, like the ten little nigger boys, gradually get less as time passes and have given an alternative, and I feel more rigid, approach by considering a steady state in the fleet when components are continuously replaced, part as they fail and part when they reach a specified life.

To ensure strict comparison with the paper, I have done a preliminary calculation by the method of Tye

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and Cundall using a failure law $(1/n)\,dn/dt = -1/(T-t)$ and find that I agree with Fig. 5 of that paper.

1. COMPARISON WITH TYE AND CUNDALL

Using the notation of the previous paper, namely:—

- r defect rate pertaining to time t
- r_0 initial defect rate of new items
- r_D average rate of replacement of defective items when life is L
- r_L average rate of replacement of items for overhaul when life is L
- r_T total replacement rate $= r_D + r_L$
- L life
- T time taken for all items to fail by defect
- t time of running of a particular item
- n number of items at time t
- n_0 initial number of items

and assuming a defect law $(1/n)\,dn/dt = -1/(T-t)$, which means that with a large number of items, they will all have failed by T, we have by the previous approach:—

$$\log n = \log(T-t) + A$$

$$n = n_0 \text{ when } t = 0.$$

Therefore $$\log n_0 = \log T + A.$$

Therefore $$A = \log \frac{n_0}{T}$$

$$n = n_0\left(1 - \frac{t}{T}\right)$$

and $$n_0 - n = n_0 \frac{t}{T}$$

Using $n_0 = 100$ and $T = 1{,}000$ this leads to a steady defect rate dn/dt of 10 per 100 hours.

Proceeding as before:—

$$U = \int_0^t n\,dt = n_0 \int_0^t \left(1 - \frac{t}{T}\right) dt$$

$$= n_0\left[t - \frac{t^2}{2T}\right]_0^t$$

$$= n_0 t\left[1 - \frac{t}{2T}\right].$$

Number of defects up to time t is

$$n_0 - n = n_0 \frac{t}{T}.$$

Therefore

$$r_D = \frac{1}{T}\left(1 - \frac{t}{2T}\right) = \frac{1}{T}\left(1 - \frac{L}{2T}\right)$$

when life is L, and

$$r_L = \frac{n}{U} = \frac{1 - \frac{L}{T}}{L\left(1 - \frac{L}{2T}\right)}.$$

So r_D/r_0 varies from 1·0 to 2·0 as L/T varies from zero to 1·0, and similarly r_L/r_0 varies from ∞ to zero, and we have the following table:—

L/T	r_D/r_0	r_L/r_0	r_T/r_0
0	1	∞	∞
0·1	1·05	9·5	10·55
0·2	1·10	4·4	5·50
0·4	1·25	1·9	3·15
0·6	1·43	1·0	2·43
0·8	1·67	0·4	2·07
1·0	2·0	0·0	2·00

which agrees well with Fig. 5 of the previous paper.

2. ALTERNATIVE APPROACH

Consider the steady state when items are replaced continuously as they fail or reach a predetermined life.

Assume that the items are replaced at small intervals of time equal to T/m, where m is large and tends to ∞ and T is as before the ultimate absolute life of individual items, and the failure law is as already assumed.

Consider any specified moment of time at the commencement of one of these intervals.

The n_0 items in use will be made up of

δn_1 with a life of 0
δn_2 with a life of T/m
δn_3 with a life of $2T/m$
δn_m with a life of $(m-1)T/m$

Rate of replacement required by failure only

Assuming for the present that only defective items are replaced as the defects occur and none is removed as "time expired."

At the end of the interval T/m the total failure will be

$$\frac{T}{M}\left[\frac{\delta n_1}{T} + \frac{\delta n_2}{T - \frac{T}{m}} + \frac{\delta n_3}{T - \frac{2T}{m}} + \frac{\delta n_m}{T - (m-1)\frac{T}{m}}\right]. \qquad (1)$$

and as the system is assumed regular and self-repeating, this sum must equal the replacement at the beginning of the interval, that is δn_1.

Using the previous failure law $dn/dt = -n/(T-t)$, we have

$$\delta n_2 = \delta n_1 \quad (1 - 1/m)$$

$$\delta n_3 = \delta n_1 \quad (1 - 2/m)$$

$$\delta n_m = \delta n_1 \quad [1 - (m-1)/m]$$

and $$\Sigma\,\delta n = n_0$$

whence $$\delta n_1 = \frac{2n_0}{m+1}. \qquad (2)$$

(When these values of δn_1 and δn_2 and so on are inserted in (1) it will be found that the total of failures in the interval T/m is δn_1 as required).

The rate of replacement (or failure) given by (2) is

$$\frac{dn}{dt} = \frac{2n_0}{m+1} \text{ divided by } \frac{T}{m}$$

or, when m is increased indefinitely,

$$\frac{2n_0}{T} \qquad (3)$$

or just twice what it would be if the items behaved

perfectly up to time T and then just expired suddenly.

Effect of controlled replacement at a given life

Now consider the effect of arbitrarily replacing all items that have not failed as soon as they attain a predetermined life of T_R.

Considering the steady state at time t, and replacement at discrete intervals of T/m as before, the n_0 items are now made up of

δn_1 with a life of 0
δn_2 with a life of T/m
δn_3 with a life of $2T/m$
δn_r with a life of $(r-1)T/m$

and $\delta n_r = \delta n_1 [1-(r-1/m]$.

Total replacements are still δn_1 made up of $(r-1)\delta n_1/m$ and δn_r.

$$n_0 = \sum_1^r \delta n_1 = r\,\delta n_1 \left[1 - \frac{r-1}{2m}\right].$$

Put $r = m/k$, which means that arbitrary replacement takes place at $1/k$ of the absolute life of the item.

Then

$$n_0 = \delta n_1 \frac{m}{k}\left[\frac{1 - \frac{1}{k}}{2m}\right]$$

leading to

$$\delta n_1 = \frac{n_0 2k^2}{m(2k-1)}$$

when m is increased without limit, and the steady rate of replacement

$$dn/dt = \text{limit } \delta n_1 \frac{m}{T}$$

$$= \frac{n_0 2k^2}{(2k-1)T} \quad . \quad . \quad . \quad . \quad (4)$$

and reduces to $2n_0/T$ when $k=1$ or no arbitrary replacement is made.

It is of interest to note that if (say) $k=4$, that is the items are arbitrarily replaced at one quarter of their absolute life, twice as many replacements are made as if all were allowed to fail, and three quarters of these replacements are of items still working.

This again demonstrates the fact that *provided it is no more expensive in the widest sense* to have a random failure than to make a controlled replacement, it is always cheaper to let the items fail naturally.

It is doubtful, however, if in practice the foregoing condition is ever true, and a random failure that may lead to a delay or an uncompleted journey must be heavily weighted against a normal routine replacement.

Calculation of the best controlled replacement life

Let us now assume that the total cost of a random failure is on the average p times as great as that of a routine replacement.

The total cost of keeping the service running with n_0 items of equipment is then proportional to

$$\delta n_r + p(\delta n_1 - \delta n_r).$$

The cost $$C = \text{constant} \times \left[\frac{(1-p)(k-1)}{k} + p\right]\delta n_1$$

$$= \text{constant} \times \frac{2k(p+k-1)}{2k-1},$$

and for any value of $p > 1$ we can choose k (defining the arbitrary replacement time) so that C is a minimum.

$$\frac{dC}{dk} = \frac{d}{dk}\left[\frac{2k^2}{2k-1} + \frac{2k(p-1)}{2k-1}\right] = \left[\frac{2[2k^2 - 2k - (p-1)]}{(2k-1)^2}\right].$$

For a minimum value of C therefore

$$2k(k-1) = p-1,$$

leading to the following figures:—

p	k	*Arbitrary replacement life* ($T = 1{,}000$)
1	1·00	1,000
2	1·37	730
4	1·82	550
6	2·16	463
8	2·43	412
10	2·68	373

Comment by W. Tye, O.B.E., F.R.Ae.S.

(Air Registration Board)

MR. STEVENS' NOTE is an interesting and valuable extension of the article by Mr. Cundall and myself. In particular I was glad to see Mr. Stevens' analysis based on the concept of continuously replacing items as they fail or reach a predetermined life. This concept is obviously close to the actual operational procedure. It differs in method from that used in the previous article, where, for ease of calculation, we assumed a specified set of items which diminished with time. So far as I can see, however, the two methods are capable of leading to the same result.

With regard to Mr. Stevens' general conclusions, I agree that if replacement of defective items is no more expensive in the most general sense than replacement of time-expired items, it always pays to wait until an item fails, regardless of the particular "law of failure" assumed. In reality, however, the cost (in the general sense) of replacing defective items is likely to exceed the cost of replacing time-expired items. For this reason the particular law of failure becomes important. If the probability of failure remains constant, irrespective of hours of use (Sections 4 and 5 of the previous paper) then it pays to wait for items to fail by defects regardless of the fact that the cost of replacing a defective item is higher than replacing a time-expired item. It is only when the law of failure assumes a rising

value with hours of use (*e.g.* a particular case being given in Section 6 of the previous paper and also by Mr. Stevens) that it does not pay to wait for defects. Depending on how rapidly the law of failure rises with hours of use, and on the cost of replacing a defective item in comparison with a time-expired item, an optimum life can be assessed.

I think it is important to realise that from such evidence as is available, the law of failure which most often fits the observations is the constant value, and not a rising value of the kind assumed in Section 6 of the previous paper. Thus it may well be that in practice the optimum life concept does not often apply. It was for this reason that Mr. Cundall and I concentrated most of our attention on the case of constant defect rate.

Stress Analysis of Circular Frames

by

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IN HIS interesting paper on the Stress Analysis of Circular Frames (March 1953 JOURNAL) K. J. Dallison discussed numerous methods of attacking the problem and mentioned that the most favoured method of the determination of the skin shear distribution at a loaded flexible frame has been to represent the shear distribution by a trigonometric series with the coefficients unknown at the beginning. The strain energy of the loaded frame and adjacent skin is then written down in terms of the assumed shear flow, and minimised with respect to each of the coefficients. This allows one to find the coefficients. Since the author did not mention the origin of this method, some remarks on that subject may be in order.

The question of determining the shear distribution in an energised system with continuously distributed stresses reduces to the problem of minimising the integral expression for the strain energy. This problem is inseparably connected with the calculus of variation and consequently with the Euler-Lagrange equation. Some applications of this equation to such problems are given in the book by I. S. Sokolnikoff and R. D. Specht; in the paper by Th. Pöschl, and in many other references. But in more complicated structures the solution of Euler-Lagrange equation presents enormous difficulties and some approximative methods must be applied. In 1908, a Swiss physicist, Walther Ritz (who died prematurely), closely connected with the Göttingen school of mathematicians, proposed a method of solving such variational problems. The main feature of his method is the representation of the solution by means of a sequence of functions with unknown coefficients (called minimising sequence). The coefficients have to be determined in the manner indicated by Dallison. Hence the method, mentioned by Dallison, and the so-called strain energy method are Ritz's methods applied to particular integrals. Ritz discussed thoroughly the convergence of the minimising sequence and pointed out that not every sequence can give a solution. For example, when the solution is an even function and the minimising sequence consists of odd functions, one can not expect a fair solution even when the series of function is a convergent one. Practically, each energy system seems to possess its own minimising sequence, which should be known from tests or from some preliminary calculations.

Lack of space does not allow the writer to discuss more throughly the reasons why the name of Ritz is forgotten with reference to his method, but he hopes that the presentation of these historical facts will be of some use to the better understanding of the methods discussed by Dallison.

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Note on the Application of Busemann's Formula to a Simple Two-Dimensional Curved Aerofoil Section

by

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THE THEORETICAL aerodynamic properties of supersonic aerofoils of polygonal section are well known. This note describes the results obtained by applying supersonic aerofoil theory to a simple curved aerofoil of arbitrary maximum thickness position, the leading and trailing edges of which are pointed.

The results for a symmetrical curved section are compared with the theoretical properties of a symmetrical double-wedge section.

1. GENERAL CURVED SECTION

In previous published work on curved sections, use has been made of straight lines faired by circular arcs. While this is satisfactory for a particular section, the

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algebra tends to become cumbersome for a general section, due to the discontinuity, and it is desirable to make use of a simple continuous function to define the profile of the top or bottom surface of the aerofoil.

Examination of various possible relations to meet the requirements for the profile suggests that the simplest is of the form

$$y = A(x - x^n)$$

where x and y are the chordwise and thickness ordinates respectively.

The constants A and n may be determined from the maximum thickness conditions as follows:—

At the point of maximum thickness, $x = x_m$, $y = y_m$ and $dy/dx = 0$.

Now $$\frac{dy}{dx} = A(1 - nx^{n-1}).$$

Hence $$nx_m^{n-1} = 1, \text{ since } A \neq 0$$

and $$A = \frac{y_m}{x_m - x_m^n}.$$

The value of n appropriate to a given maximum thickness position is best determined from a curve of n against x_m such as that given in Fig. 1.

The foregoing equations have several points of mathematical interest which affect the way in which they may be used. As the index n tends to unity, the position of the point of maximum thickness may be shown to tend to $x = 1/e$ and the slope at $x = 0$ for $n < 1$ becomes infinite, *i.e.* an aerofoil of this profile would have one end blunt if $n < 1$, corresponding to the maximum thickness position at $x_m < 1/e$. For $n = 2$, the profile is a parabola with the maximum thickness at $x = 0{\cdot}5$ about which point it is symmetrical. In view of this, a restriction that $n \geqslant 2$ will be imposed, and for sections in which the maximum thickness is farther

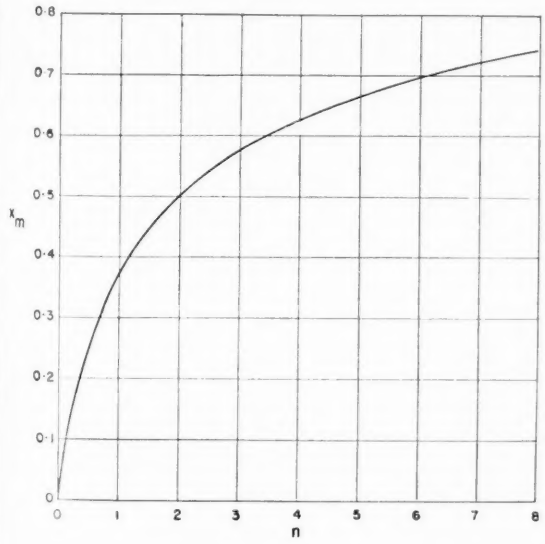

FIGURE 1.

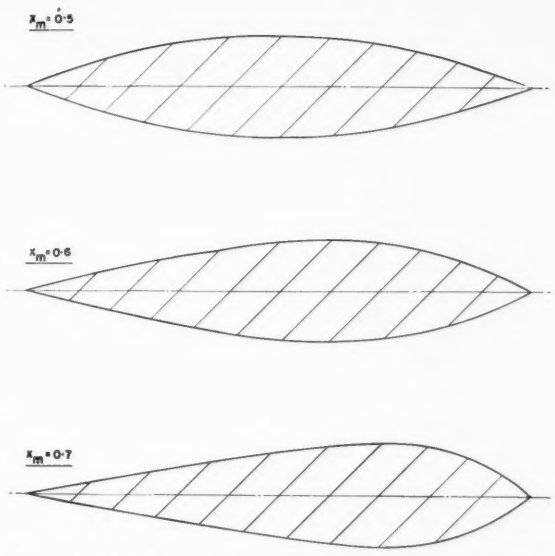

FIGURE 2. Specimen symmetrical curved supersonic sections.

forward than $x = 0{\cdot}5$ ($x = 0$ being regarded as the leading edge of the aerofoil), the relation $y = A[(1-x) - (1-x)^n]$ will be used to define the profile.

Profiles for a 20 per cent. thick aerofoil with $x_m = 0{\cdot}5$, $0{\cdot}6$ and $0{\cdot}7$ are shown in Fig. 2. The profiles for $x_m = 0{\cdot}4$ and $0{\cdot}3$ are identical to those of $x_m = 0{\cdot}6$ and $0{\cdot}7$, the leading edges for $x_m = 0{\cdot}6$ and $0{\cdot}7$ becoming the trailing edges for $x_m = 0{\cdot}4$ and $0{\cdot}3$ respectively.

The aerodynamic theory to be applied to the aerofoil is limited by the requirement that the nose angle of the section shall be less than the shock detachment angle for the Mach number at which the section is operating. The function $(1/y_m)(dy/dx)_0$ is plotted against x_m in Fig. 3, and from this the nose semi-angle θ may be obtained since

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)_0 \simeq \left(\frac{dy}{dx}\right)_0.$$

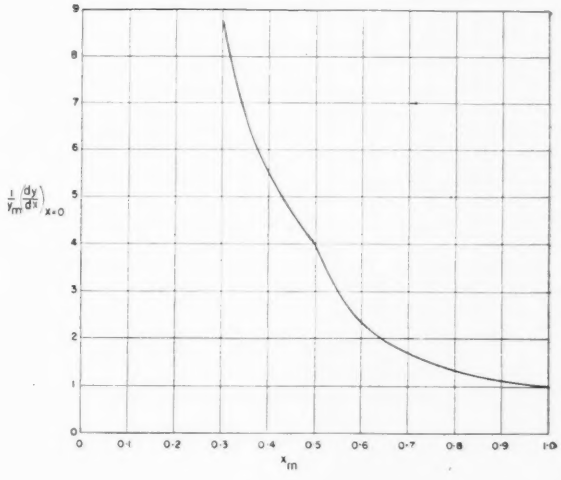

FIGURE 3.

The discontinuity in slope of this curve at $x_m = 0{\cdot}5$ should be noted; this corresponds to the change over between the equations defining the profiles.

2. AERODYNAMIC CHARACTERISTICS

Having now derived a suitably simple profile, the aerodynamic coefficients may be obtained by the method given in Ref. 1.

θ, the slope of the surface of the aerofoil is positive for compressive flow; hence for the upper surface $\theta = dy/dx$, and the integrals required for the aerodynamic coefficients become

$$I_0 = \int \theta \, dx = \int_0^1 A(1 - nx^{n-1})\, dx = 0$$

$$I_1 = \int \theta^2 dx = \frac{A^2 (n-1)^2}{2n-1}$$

$$I_2 = \int \theta^3 dx = A^3 \left(\frac{3n^2}{2n-1} - 2 - \frac{n^3}{3n-2} \right)$$

$$I_3 = \int \theta \, x \, dx = \frac{A}{2} \left(\frac{1-n}{1+n} \right)$$

$$I_4 = \int \theta^2 x \, dx = A^2 \left(\frac{n}{2} + \frac{1}{2} - \frac{2n}{n+1} \right).$$

If the maximum thickness of the aerofoil above and below the aerofoil is used to determine the coefficients A, rather than the maximum ordinate, which would be positive for the upper surface and negative for the lower, these integrals may be used for both surfaces instead of regarding $\theta = -dy/dx$ for the lower surface.

It should be noted that if the point of maximum thickness is forward of the mid-chord point and the alternative form of the equation defining the profile is used,

i.e. $\quad y = A(1-x) - (1-x)^n,$

then
$$+I_1(1-x) = +I_1(x)$$
$$+I_2(1-x) = -I_2(x)$$
$$+I_3(1-x) = +I_3(x)$$
$$+I_4(1-x) = -I_4(x) + I_1(x).$$

Omitting the terms which are zero, the coefficients of lift, drag, and pitching moment about the mid-chord point are given by:—

$$C_L = 2C_1\alpha + C_2(I_{1L} - I_{1u})$$

$$C_D = C_1(I_{1u} + I_{1L}) + 2C_1\alpha^2 + C_2(I_{2u} + I_{2L}) + 3C_2(I_{1L} - I_{1u})\,\alpha$$

$$C_{m\frac{1}{4}} = C_1(I_{3u} - I_{3L}) + C_2(I_{4u} - I_{4L}) - 2C_2(I_{3u} + I_{3L})\,\alpha + \tfrac{1}{2} C_2(I_{1L} - I_{1u})$$

where C_1 and C_2 are the first and second order Busemann coefficients respectively.

The suffixes "u" and "L" in the equations refer to the integrals for the upper and lower surfaces of the aerofoil respectively.

For an aerofoil which is symmetrical about the chord line, $A_u = A_L$ and $n_u = n_L$. Hence for this case

$$C_L = 2C_1\alpha$$

$$C_D = 2C_1 \frac{A^2 (n-1)^2}{2n-1} + 2C_1\alpha^2 + 2C_2A^3 \left(\frac{3n^2}{2n-1} - 2 - \frac{n^3}{3n-2} \right)$$

$$C_{m\frac{1}{4}} = -2C_2A \left(\frac{n-1}{n+1} \right) \alpha.$$

If the maximum thickness is forward of the mid-chord point, then the equations are

$$C_L = 2C_1\alpha$$

$$C_D = 2C_1 \frac{A^2 (n-1)^2}{2n-1} + 2C_1\alpha^2 - 2C_2A^3 \left(\frac{3n^2}{2n-1} - 2 - \frac{n^3}{3n-2} \right)$$

$$C_{m\frac{1}{4}} = -2C_2A \left(\frac{n-1}{n+1} \right) \alpha,$$

where the values of n and A are those appropriate to a maximum thickness at a point $(1 - x_m)$ in the original equation $y = A(x - x^n)$.

3. COMPARISON WITH DOUBLE-WEDGE SECTION

The variation of wave drag coefficient at zero lift with position of maximum thickness, for a series of Mach numbers for symmetrical aerofoils of 10 per cent. thickness/chord ratio is given in Fig. 4, together with the drag coefficient of a symmetrical double-wedge section of the same thickness.

The drag curves make no allowance for the skin friction or separation drags. The latter would increase the drag considerably with the maximum thickness aft, and this effect will be greatest on the curved sections, owing to the increased trailing edge angles.

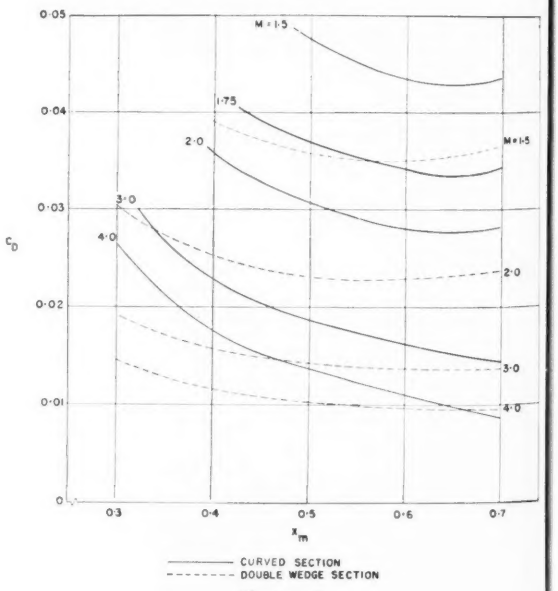

FIGURE 4.

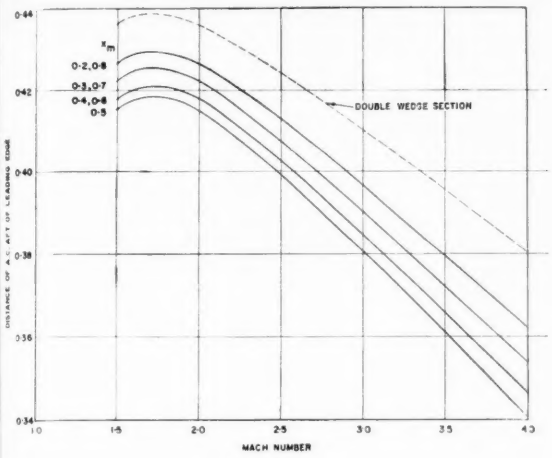

FIGURE 5.

The theory is invalid when the leading edge shock wave becomes detached and this occurs at large leading edge angles and low Mach numbers. This limit is shown approximately on the curves. Nose shock detachment will occur earlier on the curved section owing to the increased nose angle for a given maximum thickness position.

From the foregoing, it is seen that under all conditions the supersonic drag of the curved section is greater than that of the equivalent double-wedge section. The difference is greatest with the maximum thickness well forward.

The aerodynamic centre of the curved section is farther forward than that of the equivalent double-wedge section. As the maximum thickness is moved forward or aft of the mid-chord position, the aerodynamic centre moves aft. The position of the aerodynamic centre of the symmetrical double-wedge section is invariant with position of the maximum thickness. This is illustrated in Fig. 5.

ACKNOWLEDGMENT

The author wishes to thank Mrs. P. M. Measures for making many of the calculations used to illustrate this note.

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THE LIBRARY
Reviews

SERVOMECHANISM ANALYSIS. *George J. Thaler and Robert G. Brown. McGraw-Hill Electrical and Electronic Engineering Series* 1953. 414 *pp.* 64*s. net.*

This book is compiled from material which the authors have taught in undergraduate and post-graduate courses. The result is a clear, well-presented account of the methods of analysing and designing servo systems.

Certain writers in this and similar fields have an irritating tendency to over-explain the simpler mathematics while shunning the more advanced. The authors have avoided this. They assume a reasonable undergraduate knowledge of mathematics, together with a familiarity with the basic principles of electrical components. Necessary mathematical work, though occasionally a little clumsy, is then full enough to make the book self-contained. For fuller treatment and further developments the reader is referred to the bibliography, containing over 200 references, which forms one of the appendices.

Worked examples are plentiful and obviously chosen with a good knowledge of the difficulties usually encountered by students. Most of the chapters conclude with a set of problems to be worked by the reader.

The Laplace Transformation is first presented as the mathematical tool best suited for analysis. The treatment is standard and the usual theorems are given.

The equations of physical systems follow, in which mechanical, electrical, thermal and hydraulic systems are discussed. The replacement of these systems by electrical analogue networks is described in detail.

A chapter on transient analysis deals fully with the proportional-error arrangement, and then considers the effect of derivative and integral control.

The transfer-function concept leads naturally to the use of polar plots of transfer function and frequency response. The Nyquist criterion is developed clearly, and at adequate mathematical length; in this connection, the authors' use of the term "real root" of a polynomial equation to include the real part of a complex root is unfortunate. The useful and convenient Root Locus method of deducing transient response is also described.

In the section devoted to the design of servo systems, a comprehensive examination is made of the effects of gain adjustment, series and parallel compensation, followed by a chapter on the use of logarithmic plotting in analysis and design.

An interesting feature is the inclusion of a chapter on relay servo-mechanisms. No attempt is made to present an exact analysis, but practical engineering approximations are used to give a reasonable compromise between accuracy and simplicity of treatment.

To avoid breaking the thread of the main discussion the working of the actual components of servo systems is explained in more detail in a series of appendices.

In spite of its formidable price, this lucid and well-arranged exposition of the theory of servo-mechanisms is highly recommended.—S. J. GARVEY (Vickers-Armstrongs, Supermarine).

HANDBOOK OF STATISTICAL METHODS IN METEOROLOGY. *C. E. R. Brooks and N. Carruthers. Meteorological Office, Air Ministry. H.M.S.O.* 1953. 412 *pp.* 75 *figures.* 25*s. net.*

This work is intended as a textbook and reference book for those wishing to apply statistical methods to meteorological data, but it is a good deal more in that every one of the 22 chapters is concerned primarily with a statistical problem or technique, meteorological data being mainly used to provide the illustrative material. Moreover, the subject matter covers practically the whole of modern applied statistics, so that while meteorologists may be expected to benefit most from the book, the reviewer will be surprised if many other scientists and engineers do not avail themselves of its publication. And this in a field in which some choice is already available in published texts.

The book is in three parts: the distribution of observations (9 chapters), related variables (6 chapters) and time series (7 chapters). Proofs of well-known theorems are not given but the basis of new work, particularly on problems of persistence, is adequately discussed. There are eleven appendices giving important formulae, tables of functions, symbols, etc., and each chapter ends with a set of examples, with occasional hints for working. An important feature, for aviation as for meteorology, is a chapter devoted to vector variables, wind velocity being mainly in mind in the discussion. The treatment is effectively limited to two-dimensional vectors of which the components are independent and of equal standard deviation, properties which are possessed approximately by the horizontal component of the wind of the free atmosphere. There are also useful chapters on smoothing and interpolation, and on curve fitting.

Many attempts have been made to unearth periodic phenomena, other than diurnal and seasonal variation, in meteorological series, and in geophysical observations generally. Such analysis is beset by many difficulties and traps for the unwary and the authors are to be congratulated on the way they carry the reader through the territory. This part of the book, the last 46 pages, should be useful to workers in many fields. It invites comparison with the final chapters of Volume II of Kendall's *Advanced Statistics*, and is indeed a valuable complement to that study. As a practical handbook it might, however, have given a little more attention to the various methods of computing periodograms and correlograms.

The book is nicely produced on the whole and is well illustrated, although the mathematics appears rather crowded. There can be few books published today presenting such good value to the buyer and all concerned are to be congratulated on a successful enterprise.—P. A. SHEPPARD.

AIRCRAFT PROPULSION. THEORY AND PERFORMANCE. *A. W. Morley. Longmans, Green,* 1953. 221 *pp.* 115 *figures.* 28*s. net.*

Quite a few books have now been written on gas turbines for aircraft propulsion and the field is becoming more competitive for authors. This new book will be particularly appreciated by those mainly interested in the performance of aircraft engines rather than in their design and construction. The book begins with a useful chapter on the thrust required for flight and, after dealing with the basic thermodynamics and gas dynamics, proceeds to consider the characteristics of performance of the turbo-jet engine, the propeller turbine and the compound gas turbine. A chapter is also included on the piston type aero-engine and brief accounts are given in the last chapters of the ram-jet and the rocket. In each case the

treatment is by means of the parameters, usually non-dimensional, which determine the performance of the whole power plant and of its components. Examples are given showing how the performance may be calculated under different conditions. The basic aerodynamics of axial flow theory is also given, with application both to compressors and turbines.

The book is in some respects an unusual mixture. For instance, it discusses heat exchangers, which are not used in any gas turbine aircraft today, but omits entirely the centrifugal compressor. Again, although it considers the ram jet and the rocket, little attempt is made to compare the performance of these motors with jet engines and propeller turbines and thus to show the overall picture of the fields of application of different power plants. Space might perhaps have been used for this purpose at the expense of some of the basic thermodynamics, the treatment of which is quite conventional and is to be found in many other places, even assuming, as is unlikely, that the reader interested in the performance and characteristics of gas turbines, does not already know his fundamentals. The omission of almost all reference to mechanical engineering features, combustion chamber design, and materials of construction, which limit performance, is deliberate and avoids making the book too bulky, as would be inevitable if it were to approach a complete treatise on the aircraft gas turbine.

The book will be of great value both to aircraft and engine designers and to teachers. The author has clearly had experience of teaching the subject to aircraft engineering students and his method of presentation is clear and logical. Altogether an excellent book for those who are primarily interested in the performance and general characteristics of the various present day types of aircraft power plants.—O. A. SAUNDERS.

THEORY OF ELASTICITY AND PLASTICITY. *H. M. Westergaard. New York, John Wiley; London, Chapman & Hall* 1952. 176 *pp. Diagrams.* 40*s. net.*

Although the author's untimely death forestalled the completion of the work as originally planned, the book as published does in fact form a reasonably connected whole and can be warmly recommended as an introductory treatment of elastic theory suitable for post-graduate students of engineering.

An unusual and noteworthy feature of the book, which might profitably be followed by writers in other branches of applied science, is a historical chapter, which is used to introduce many of the topics subsequently discussed in detail. These include stress, strain, Hooke's law, the fundamental equations of elasticity, the laws of plasticity, strain potential, hollow cylinders and spheres, thermal stresses, the Galerkin vector, and effects of a single force. The main omission is the treatment of two-dimensional elastic problems; and the inclusion of "Plasticity" in the title is misleading in that only a very small proportion of the work is actually devoted to this subject.

Although the book is written from an engineering standpoint and is intended for the use of research students working in the fields of mechanical or civil engineering, the treatment of the subject, which involves at least as much mathematics as the average engineering research student has at his disposal, will also be of interest to applied mathematicians working on elastic problems.

The book is lucidly written, well referenced, and excellently produced. The price is on the high side, although this is doubtless an unavoidable result of American publication.—D. M. A. LEGGETT.

SOLUTION OF PROBLEMS IN STRENGTH OF MATERIALS. *S. A. Urry. Pitman. London* 1953. 381 *pp. Diagrams.* 20*s. net.*

In his preface the author states that "this book is intended for students who are preparing for an examination in Strength of Materials." He also claims that "no previous knowledge of the subject is assumed and the problems are chosen so that the theory is developed in a logical manner." The implication is that this book is by itself an adequate textbook for the study of the subject to the appropriate standard. This may be true for the student who desires no deeper knowledge than is necessary to provide him with the required facility in solving examination questions. Generally, however, this book should be regarded as a supplement to a lecture course or to a more orthodox textbook, in which the fundamental theory and assumptions are discussed more fully. For this purpose, Mr. Urry has produced a very useful book, providing a wide range of fully worked examples together with further practice examples with answers.

The general standard is that of the Higher National Certificate in Mechanical Engineering and of the examinations of the various engineering Institutions, and the scope of the book can best be indicated by the following list of the chapter headings: Simple stress and strain, elasticity; Shearing forces and bending moments; Longitudinal stresses in beams; Torsion of circular shafts; Deflection of beams; Propped beams and cantilevers, encastré beams; Struts; Complex stresses; Principal strains and the elastic constants; Thin shells; Thick cylindrical shells; Strain energy; Springs; Shearing stresses in beams; Appendix I, Analysis of experimental results; Appendix II, Moments of area.

Although propped beams are considered from the point of view of compatability of deflections, no mention is made of continuous beams. It may be this omission which leads the author to define a simply supported beam as "one which rests on *two* (reviewer's italics) knife edges." It would be better, surely, to relate this definition to the nature of the support without specifying the number of supports, even though the chapter in which the definition occurs is concerned only with statically determinate beams.

From the point of view of the aeronautical student the treatment of struts is inadequate, only the Euler formula and Rankine's formula being considered. No attention is given to the effects of eccentricity of load or of initial bow, nor are there any examples relating to laterally loaded struts.

In spite of these omissions the ground covered is considerable, and the layout of the worked solutions is very good. The author rightly draws attention to the need for a check on units, and inserts the units at all stages in his solutions. The notation conforms to British Standard 560 (Engineering Symbols and Abbreviations), but there may be a danger of confusion between t for tons, t for hoop stress and t for wall thickness. In fact, on page 279 the symbol t appears with both its meanings.

Although written primarily for the mechanical engineering student, the book can be recommended to those students preparing for the Strength of Materials paper in Part I of the Associate Fellowship Examination of the Society. Examiners in this subject will certainly be grateful to Mr. Urry if the publication of his book results in the adoption by candidates of the methods of setting out solutions which he has illustrated.—R. TATHAM (Saunders-Roe Ltd.).

FLYING SAUCERS. *Donald H. Menzel. Putnam. London* 1953. 319 *pp. Illustrated.* 21*s. net.*

Dr. Menzel is Professor of Astrophysics at Harvard University. His book on flying saucers makes far less entertaining reading than some earlier efforts by writers convinced that this celestial crockery is an interplanetary fleet manned by little men from Mars, Venus and all points west of the Milky Way.

However, it is undoubtedly far more useful; the author states that " Flying Saucers: (1) Do exist; (2) Have been seen; (3) Are not what people thought they saw." Any reader sufficiently interested to plod through his lengthy treatise (which is extremely thorough, sometimes slightly smug, but only very rarely questionable in its authority) should have a very good idea of what they *might* see.

That is to say, after disposing of the many " sightings " due to hoaxes and hysteria, there are a great many due to mistaken identification of aircraft, balloons, and other airborne objects, and then still others associated with meteors, other heavenly bodies, auroral effects, and optical mirage phenomena. Radar, like the human eye, can also sometimes be unreliable in its " seeing "; as regards the mirages caused by atmospheric refraction, Professor Menzel describes some simple and interesting laboratory experiments which produce plausible small-scale saucers. He also discusses the many " sightings " going well back into history; in the early years of this century, they were sometimes thought to be airships—spaceships were then not yet a familiar popular conception.

Professor Menzel, in one of his chapters, concedes that interplanetary travel is likely to be achieved by mankind within the not-too-distant future, and even that the earth might, at any time, itself receive extra-terrestrial visitors. However, like this reviewer, he regards it as very unlikely that the flying saucers are to be explained on these grounds. —A. V. CLEAVER.

BRITISH WAR PRODUCTION. *M. M. Postan. H.M.S.O. and Longmans* 1952. 512 *pp.* 32*s.* 6*d. net.*

In this book, which is part of the official history of the war, Professor Postan, an eminent economic historian, tells the story of the progress and problems of British munitions production from the lean years of the inter-war years to the peak output of 1943 and 1944 and then the demobilisation of 1945. It is a general review and is to be followed later by more specialised studies. Professor Postan is quite rightly not content merely to tell the narrative of this great achievement, but at every stage he criticises and analyses the methods of planning, Government control and the organisation of industry. He tries to discover the causes of failure and the secrets of successful effort. This is inevitably a daring and risky venture and some of those who took part in the great production struggle will disagree with many of his conclusions.

It must be remembered that this is a story and commentary based largely on the records of Government Departments and it portrays the events and problems mainly as seen through the eyes of the civil servant. The industrialist who ran the factories will find here little explanation of the problems and difficulties he had to face, the desperate expedients he often had to use, and the problems of Government control and planning as seen from the receiving end.

In this story aircraft naturally occupy a prominent place. There is a detailed discussion of the progress of aircraft production, an analysis of the causes of the invariable inability to fulfil any of the programmes 100 per cent. I think that Professor Postan does not altogether realise how much this fall-down was due to the industry's unwillingness to set itself an easy task which it knew it could achieve. It always wanted to aim at the maximum. I know from my own experience that no firm ever achieved its programme 100 per cent. without immediately asking for its programme to be increased. There is a fascinating discussion of the problems of design and technical development (pp. 322-339), but the reasons why some aircraft were failures while others, for example the Lancaster and Mosquito, were winners, still remains something of a mystery. Professor Postan seems to think that more " industrial planning " by M.A.P. of the type started by Sir Ernest Lemon at the Air Ministry would have improved the efficiency of the industry; but I expect that the thought of this, even in retrospect, will send a cold shiver down the back of many an aircraft manufacturer.

In a general review of this kind the final product, the aircraft, is in the centre of the picture. There is little discussion of the great achievements and problems of engine, propeller, undercarriage and other equipment production. That story, in my view just as fascinating and enlightening, remains to be written.

It is clear that this book is compulsory reading for anyone interested in the aircraft industry, its problems and especially its relations with Government.—E. DEVONS (The University, Manchester).

FATIGUE OF METALS. *R. Cazaud. Translated from the French. Chapman & Hall. London.* 334 *pp. Diagrams and photographs.* 60*s. net.*

Some 124 years ago, the German mining engineer, Albert, investigated the effect of repeatedly loading mine-hoist chains. His experiments, though primitive, were far reaching in result. They made clear for the first time that repeated stresses, low in comparison with the ultimate tensile stress of the material to which they were applied, could give rise to fracture. Thus dawned consciousness of the fatigue of metals, a subject whose significance grows, particularly in those fields where economy in weight is of first importance.

Towards the middle of last century, the British Government was seriously disturbed by the frequency of railway accidents. These, in many cases, appeared to be related to the deterioration during the use of the metal employed both in the construction of bridges and of rolling stock. As a result, a commission was appointed—" to enquire into the application of iron to railway structures." Its report, in the preparation of which a considerable part was played by E. A. Hodgkinson, can be rightly regarded as one of the major foundation stones of the vast body of literature relating to the fatigue of metals.

In recent years, there have been attempts to make a bibliography of works dealing with this subject. In 1941, the Battelle Memorial Institute published its excellent work *Prevention of Failure of Metals under Repeated Stress*: the 1949 edition cited 914 publications. Another bibliography made some four years later by the Australian Division of Aeronautics, gave approximately the same number of references. Thus it is clearly impossible for the average engineer or metallurgist to read, *in extenso*, all that has been written about fatigue, though admittedly its importance becomes greater and greater every day, particularly in the aeronautical field.

Thus it is entirely fitting that from time to time the current state of knowledge should be reviewed by someone having special knowledge in this field. In the past, there have been some notable examples of this. Gough's splendid work *The Fatigue of Metals* was first published in

1820. Moore and Kommer's book having the same title came out in the United States in the year following. The Battelle publication was published in 1941 and, among other works, there are two well reported symposia, one held at Melbourne in 1946 and the other at the Massachusetts Institute of Technology in 1950.

We are now offered *Fatigue of Metals* by Dr. R. Cazaud. This gives promise of being an appreciation of contemporary thought with regard to its subject, but examination shows this not to be entirely the case. The work appeared in French in 1948 and now, almost five years later, has been translated into English. We are assured that the author has taken the opportunity to add much new material. Without in any way questioning this, the book cannot be regarded as being either sufficiently comprehensive or up to date.

The significance of dislocations in crystal structure in relation to the development of slip bands is ignored. Nothing is said as to what extent recrystallisation can eliminate the effect of earlier cyclic stresses. What is stated with regard to stress concentration ignores both Inglis and Neuber, surely in the latter case an omission of consequence and not one to be too readily overlooked since one of his important contributions was made in 1937. One searches in vain for any discussion of the Cumulative Damage Rule. This was propounded by M. A. Miner as long ago as 1945. Subsequently it was elaborated by many others and in 1950, was the subject of a careful review by Professor N. M. Newark. Similarly the influence of cyclic stresses on the transition temperature of ferritic steels, clearly expounded by Professor C. W. MacGregor in 1950, finds no place in this publication of 1953. Finally, aeronautical engineers will regret the failure to draw attention to G. Forrest's important work of 1946, showing, *inter alia*, how residual stresses produced by heat treatment, or by other means, in notched light alloy test pieces, can most favourably influence their fatigue characteristics.

The foregoing are faults, but by no means the only faults of omission. To mention just two others—British aircraft engineers will be disappointed to find that Dr. Cazaud does not deal with the fatigue characteristics of the alloys of which the more important components of contemporary airframes are made—aeronautically he is of L1, L3, L39 period. He altogether ignores the more serious fatigue problems of such alloys as D.T.D.363 and 683. Further, while he does not wholly neglect the influence of temperature on the fatigue and notch fatigue characteristics of a few materials, he considers this over a thermal range far more restricted than that to which contemporary aircraft are subjected.

To sum up—this work, with its all-embracing title, does not wholly discharge its promise. Its author appears to lack sufficient metallurgical knowledge for the task he has undertaken.—P. L. TEED.

PRINCIPLES OF RADAR. *J. F. Reintjes and G. T. Coates. McGraw-Hill, London* 1953. 985 *pp. Illustrated.* 55*s.* 6*d. net.*

The original form of this work was prepared by members of the staff of the Massachusets Institute of Technology for use as a reference text in the Institute's Radar School which was established in 1941. The book was first issued to students of the School as a classified document in 1944. On account of the rapid development of radar during the war years, it was necessary to revise the book before releasing the material for general publication towards the end of 1946. The present volume is the so-called "third edition" prepared by Professors J. F. Reintjes and G. T. Coates of M.I.T., and comprises the British re-issue of the first appearance of the work in normal printed format. The book is essentially a detailed descriptive treatment of the technique of radar prepared for the use of students in the armed services of the United States of America. The subject matter is, however, by no means confined to military objectives: and the general treatment adopted makes it of equal interest in connection with the several civil applications, such as aids to surveying, navigation and meteorology, which have been developed since the end of the war.

Although some reference is made to equipments working on a wavelength of about two metres, the major portion of the material refers to the use and operation of radar installations on centimetre wavelengths. The first half of the book describes the circuit techniques used for the generation of pulses of the desired shape and repetition frequency, the use of these to modulate the radio frequency carrier waves, and the receiving and indicating equipment for the appropriate display of echo pulses on the screen of the cathode ray tube. Illustrated descriptions are given of the different forms in which the information may be presented as to the range, azimuth and elevation of the target under observation. These vary from the simple original type of "A" display, giving range only, to the now well-known Plan Position Indicator which presents a general view in polar co-ordinates of a whole field of observation. A chapter which may be useful in connection with other applications is that describing the servo-mechanisms used to control the motion of the radar antennæ, and to maintain synchronism between these and the associated rotating coil of a plan position indicator.

The second half of the book deals with the various component parts of radar installations which operate at radio frequencies, mainly in the part of the spectrum between 1,000 and 30,000 Mc/s (wavelengths 1 to 30 cm). Chapters are devoted to the special valves—triodes, klystrons and magnetrons—used to generate these ultra-high frequency oscillations, and to the transmission lines, wave guides and antenna systems forming the essential links through which the outgoing exploring pulses are sent and the incoming echo pulses are received. A short chapter on propagation describes the main features concerning the reflection of the waves from land and sea, and the effect of the atmosphere on their transmission.

The book is essentially descriptive in nature and no mathematics beyond trigonometry and simple differentiation and integration are used. The text is very clearly presented as are also the 650 diagrams, many of which are of exceptional quality and interest. There are occasional footnotes giving references to more specialised papers on some portions of the subject. With one or two exceptions, these references are to American literature; and this opportunity may be taken of drawing attention to the large amount of British scientific and technical literature which has now been in existence for several years (*see* for example, *Journal of the Institution of Electrical Engineers*, London, 1946, Vol. 93A, Part IIIA). On the other hand, it is clearly an advantage to have the large mass of technical information and experience which is scattered throughout the proceedings and journals of societies, assimilated, digested and collected together in a single volume. While not every student will want to carry about with him a book of nearly 1,000 pages and weighing about four pounds, there is no doubt that the present volume is a most useful encyclopædic work of reference, which all students and workers in this subject should have at their disposal.—R. L. SMITH ROSE.

DIE GASTURBINE. *J. Kruschik. Springer, Vienna* 1952. 469 *pp. Diagrams.* £5 7*s. net.*

The postwar years have witnessed a steady stream of books dealing with varying degrees of thoroughness with the descriptive aspects of the gas turbine in its various forms, usually supported by a certain amount of theoretical discussion of the fundamental operating cycles. The present volume is no exception, and for the English-speaking reader must necessarily suffer from being written in German, and from its relatively late appearance. Within these limitations, however, the author has produced a typically painstaking digest of the available press releases on power plants for aircraft and other applications, together with a considerable amount of theoretical background.

After a concise historical introduction and review of the major variants of the basic constant pressure cycle, the specific outputs and cycle efficiencies of these variants are dealt with in laborious detail, concluding with a useful set of curves showing the variation of these quantities with pressure ratio and turbine inlet temperature. A number of temperature-entropy and allied diagrams are then introduced: these permit calculation of the compressor, combustion chamber and turbine overall performance, and their use is suitably illustrated by numerical examples.

The major components—compressor, combustion chamber, turbine, fuel system and heat exchanger—are then described in full physical detail, though with negligible discussion of theoretical aspects. This is followed by a discussion of part-load specific performance, a section which leans heavily on Mallinson and Lewis's well-known paper. As is all too often the case, the reader is given little or no help with the all-important problem of how to predict the part-load performance of a given plant.

The latter half of the book is essentially a summary of published design and performance details of numerous gas turbine engines of all types, supplemented by a valuable collection of data on heat resistant and other structural materials.

To summarise, this book presents a very adequate survey of the field which would be useful as an introduction for intending specialists or for the more enlightened potential user of gas turbines, although it is really too detailed for this purpose. On the other hand, it has little to offer the practising gas turbine engineer, omitting as it does most of the information required on such topics as blading design, stressing and detailed performance calculation.—J. R. PALMER (College of Aeronautics).

MOTEURS A RÉACTION. *G. Lavoisier. Technique et Vulgarisation, Paris* 1952. 232 *pp. Diagrams.* 1,340 *fr. net.*

The author has prepared a useful pocket-sized introduction to the theory and application of aircraft jet propulsion. A brief introductory survey of the principles and history of the reaction motor is followed by a more detailed study of elementary turbo-jet theory. A short chapter is devoted to the design and construction of aircraft gas turbines, dealing briefly with the centrifugal compressor, but giving in some detail the elements of axial compressor and turbine design. The mechanism of combustion is described, with notes on the main types of combustion chamber, and mention is made of the problems of engine control, lubrication and cooling. Considerable space is given to a description of the Rolls-Royce/Hispano-Suiza Nene and the S.N.E.C.M.A. Atar 101, as representative current types, and a few details are given of the Russian M 012 (Jumo 004). The only other turbo-jet engines mentioned are the Armstrong-Siddeley/Curtiss-Wright Sapphire and the Russian M 018 (B.M.W. 003), although a few details are given of the S.N.E.C.M. T.B. 1000 and Pratt and Whitney T 34 turbo-prop engines. About a third of the book is devoted to a general review of the gas turbine, beginning with a recapitulation of the concept of energy and entropy, and going on to discuss cycle and component efficiencies and their controlling factors. Various methods of increasing power output are discussed: cooled blades, water and water/methanol injection, reheat, by-pass and compound engines, and brief mention is made of the supersonic compressor. Ram-jets, pulse-jets and rockets are all reviewed, and the chapter concludes with some comments on the future application of the gas turbine. Finally, there are some notes on the maintenance of aircraft turbine engines.

In common with similar contemporary publications, the book suffers from the restrictions of military security, most of the references being to the Nene or to the Jumo and B.M.W. engines. As a general introduction to the subject for students and engine users, it is, however, satisfactory, although the description of the phenomenon of compressor surging (p. 45) is somewhat hazy and liable to be misleading. There is a statement (p. 50) that the Nimonic alloys cannot be rolled into sheet and fabricated, and there are one or two small errors such as a reference to the Rolls-Royce Goblin. The maintenance notes are too simplified to be of much practical value, but serve to give the general reader some idea of the operations involved. On the whole, an adequate introduction to jet propulsion.—ALAN H. ROBINSON (National Gas Turbine Establishment).

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** Items marked with an asterisk may not be taken out on loan.*

Reports

AERODYNAMICS

Boundary Layer

See also Wings and Aerofoils

Boundary layer flow along a flat plate with uniform suction. J. M. Kay. R. & M. 2628 (*May* 1948, *published* 1953).

Experiments have been made in the closed-circuit wind-tunnel at Cambridge University to determine the effectiveness of distributed suction as a means of controlling and stabilising the flow in a boundary layer. These experiments have shown that the laminar exponential suction profile can be established and retained, provided that the boundary layer is in an undisturbed laminar condition at the start of the suction region. The variation with rate of suction of the total effective drag of a flat plate has also been investigated.—(1.1.5).

Näherungsverfahren zur Berechnung turbulenter Grenzschichten unter Benutzung des Energiesatzes. J. Rotta. Mitteilungen Nr. 8. *Max-Planck-Institut für Strömungsforschung, Göttingen* (1953). (1.1.4).

Control Surfaces

See also Stability and Control

The aerodynamic characteristics of flaps. A. D. Young. R. & M. 2622 (*February* 1947, *published* 1953).

This report collects and summarises the results of work that has been done both in this and other countries on the aerodynamic characteristics of flaps both before and during the 1939-45 War.—(1.3.4).

The microstructure of turbulent flow. A. M. Obukhoff and A. M. Yaglom. N.A.C.A. Technical Memorandum 1350 (*June* 1953). (*Original publication* 1951.)

An attempt is made to describe quantitatively the structure of the velocity, pressure, and acceleration fields for all scales for which the theory of Kolmogoroff is applicable.—(1.3).

Fluid Dynamics

Les autoplissements et les équations intrinsèques des fluides visqueux. F. J. Bourrières. Publications Scientifiques et Techniques du Ministère de L'Air. No. 279 (1953). (1.4).

Internal Flow

A visualization study of secondary flows in cascades. A. G. Hansen, H. Z. Herzig and G. R. Costello. N.A.C.A. Technical Note 2947 (*May* 1953).

Flow-visualisation techniques are employed to ascertain the streamline patterns of the non-potential, secondary flows in the boundary layers of cascades, thereby providing a basis for more extended analyses in turbomachines.—(1.5.4).

Subsonic flow of air through a single-stage and a seven-stage compressor. Chung-Hua Wu. N.A.C.A. Technical Note 2961 (*June* 1953).

A method recently developed for solving the steady flow of a non-viscous compressible fluid along a relative stream surface between two adjacent blades in a turbomachine is applied to investigate the low- and high-speed subsonic air flow through a single and through a seven-stage axial compressor. Appreciable radial flow results from the radially increasing values of the angular momentum of the air particles associated with the "symmetrical velocity diagram at all radii" used in the examples. The axial-velocity distribution obtained for the example checked well with a simple approximation solution obtained previously by assuming simple sinusoidal radial-flow paths.—(1.5.2)

Loads

The drag of finite-length cylinders determined from flight tests at high Reynolds numbers for a Mach number range from 0·5 *to* 1·3. *C. J. Welsh. N.A.C.A. Technical Note* 2941 (*June* 1953).

Results of a free-flight investigation to determine the drag of circular, finite-length cylinders are presented for a Mach number range from about 0·5 to 1·3. Also included are drag results of previous experimental tests of infinite-length cylinders. Drag of circular cylinders at supersonic speeds is largely independent of fineness ratio and Reynolds number; whereas, at subsonic speeds, the drag of finite-length cylinders (fineness ratios of about 60 and below) increases as their fineness ratios increase.—(1.6).

Stability and Control

See also Aeroelasticity

Effects on longitudinal stability and control characteristics of a Boeing B-29 airplane of variations in stick-force and control-rate characteristics obtained through use of a booster in the elevator-control system. C. W. Mathews, D. B. Talmage and J. B. Whitten. N.A.C.A. Report 1076 (1952).

The longitudinal stability and control characteristics of a B-29 aeroplane have been measured with a control surface booster incorporated in the elevator-control system. The measurements were obtained with the booster operating to provide various control-force gradients and various maximum rates of control motion. Results are presented which show the effect of these booster parameters on the handling qualities of the test aeroplane. (Formerly Technical Note 2238.)—(1.8).

Report on the special field "interference" to the wind-tunnel committee in February 1945. *H. Schlichting. N.A.C.A. Technical Memorandum* 1347 (*May* 1953).

Contains an outline of investigations dealing with interference effects on the static stability of various aeroplane configurations that had been conducted, were being conducted, and were started at the time of this present report. Results of several investigations are presented and discussed briefly. A supplement is attached showing the configurations tested and outlining the various test programmes. Suggestions for future investigations are also included.—(1.8).

Method for calculating the rolling and yawing moments due to rolling for unswept wings with or without flaps or ailerons by use of non-linear section lift data. A. P. Martina. N.A.C.A. Technical Note 2937.

The methods of N.A.C.A. Reports 865 and 1090 have been applied to the calculation of the rolling- and yawing-moment coefficients due to rolling for unswept wings with or without flaps or ailerons. The methods allow the use of non-linear section lift data together with lifting-line theory. Two calculated examples are presented in simplified computing forms in order to illustrate the procedures involved.—(1.8.1).

Investigation of lateral control near the stall: flight investigation with a light high-wing monoplane tested with various amounts of washout and various lengths of leading-edge slot. F. E. Weick, et al. N.A.C.A. Technical Note 2948 (*May* 1953).

Flight tests were made with a light high-wing monoplane to investigate possibilities for obtaining satisfactory lateral control at low flight speeds. The aeroplane was tested with a plain straight wing, with the wing twisted to various amounts of washout, and with leading-edge slots covering various portions of the span. With each wing configuration the tests were repeated with power on and off and for three centre-of-gravity locations. The flow over the wing was observed by means of tufts attached to the upper surface. The results of the tests indicate that reliable control at angles of attack very near the stall can be achieved by the proper use of leading-edge slots and wing washout.—(1.8.1 × 1.3.5).

NOTE:—*The figures in parenthesis at the end of each Summary are for office use only.*

THERMO-AERODYNAMICS

Internal-liquid-film-cooling experiments with air-stream temperatures to 2,000°F. in 2- and 4-inch-diameter horizontal tubes. G. R. Kinney, A. E. Abramson and J. L. Sloop. N.A.C.A. Report 1087 (1952).

An experimental investigation of internal-liquid-film cooling was conducted in 2- and 4-inch-diameter straight metal tubes with air flows at 600° to 2,000°F. and Reynolds numbers from $2{\cdot}2$ to 14×10^5. The coolant was water at flows of 0·8 to 12 per cent. of air flow. Visual observations of liquid-film flows were made in transparent tubes with air flows at 80° and 800°F. and diameter Reynolds numbers from $4{\cdot}1$ to 29×10^5. Flows of water, water-detergent solutions, and aqueous ethylene glycol solutions were investigated. Liquid-coolant films were established and maintained around and along the tube wall in concurrent flow with the hot air. The tube wall was kept below the boiling temperature of the coolant over the surfaces covered by liquid coolant.

Heat transfer to bodies in a high-speed rarefied-gas stream. J. R. Stalder, G. Goodwin and M. O. Creager. N.A.C.A. Report 1093 (1952) (*formerly T.N.* 2438).

The equilibrium temperature and heat-transfer coefficients for transverse cylinders in a high-speed stream of rarefied gas were measured over a range of Knudsen numbers from 0·025 to 11·8 for Mach numbers from 2·0 to 3·3. Fully developed free-molecule flow was found to occur first at Knudsen numbers of approximately 2·0. The Nusselt number was found to be a function of the Reynolds number only, and the temperature-recovery factor to depend primarily on the Knudsen number.—(1.9.1).

Pressure distributions about finite wedges in bounded and unbounded subsonic streams. P. L. Donoughe and E. I. Prasse. N.A.C.A. Technical Note 2942 (*May* 1953).

An analytical investigation of incompressible flow about wedges was made to determine effects of tunnel-wedge ratio and wedge angle on the wedge pressure distributions. The region of applicability of infinite wedge-type velocity distribution was examined for finite wedges. Theoretical and experimental pressure coefficients for various tunnel-wedge ratios, wedge angles, and subsonic Mach numbers were compared.—(1.9.1).

WINGS AND AEROFOILS

See also TESTING AND INSTRUMENTS

The calculation of lift taking account of the boundary layer. J. H. Preston. R. & M. 2725 (*November* 1949, *published* 1953).

The purpose of this paper is to find a sound approach to the problem of the theoretical prediction of sectional characteristics taking account of the boundary layer. Attention is mainly concentrated on the lift, since it is on the accuracy of this calculation that the accuracy of calculations for other characteristics such as pressure distribution and moments must depend. Calculations of the lift and of the velocity at the edge of the boundary layer near the trailing edge have been made for two dissimilar symmetrical aerofoils at an incidence of 6 deg., using boundary-layer data taken from experiment. The method of calculation satisfies the fundamental theorem that no net vorticity is discharged into the wake at the trailing edge and in contrast to the earlier calculations of R. & M. 1996, full account is now taken of the effect of the boundary layer on the velocity field outside the boundary layer, so that the empiricism of that report is avoided. The present calculations harmonise the two different methods of approach which have been used in the past, namely, the one in which the loss of lift below the Joukowski value was attributed entirely to the incidence and camber effects of the boundary layer, and the other in which the vorticity theorem was satisfied, but boundary-layer camber effects were ignored.—(1.10.1.1 × 1.1).

An approximate method of calculating the subsonic velocity distribution on high-aspect-ratio swept wings of small thickness at zero lift. F. Keune. KTH, Sweden, Aero TN 26. (*February* 1953.)

The discussion deals with the influence of aspect ratio on the flow around high-aspect-ratio wings. To the well-known strip method giving the two-dimensional profile flow at any point on the span, two terms must be added which correspond to the analogous theory for low-aspect-ratio wings. The kink effect, which is given for general wing form and thickness distribution, is equivalent to the exact value given by S. Neumark when the special case of a rectangular wing with constant spanwise thickness distribution is considered.—(1.10.1.2).

*Theorems concerning the drag reduction of wings of fixed planform. A. M. Rodriguez, P. A. Lagerstrom and E. W. Graham, Douglas Report No. SM-*1445. (*March* 1953.)

The paper deals with the problem of determining the geometry (camber, twist, and mean angle of attack) which leads to minimum drag of a wing of fixed planform and given lift in supersonic flow. The linearised theory of an inviscid fluid is used.—(1.10.1.2).

*Some effects of streamwise gaps on the aerodynamic characteristics of low-aspect ratio lifting surfaces at supersonic speeds. Z. O. Bleviss and R. A. Struble. Douglas Report SM-*14627. (*April* 1953.) (1.10.1.2.)

*Drag reduction due to wing incidence and twist for wing-body combinations. B. Beane. Douglas Report SM-*14740. (*March* 1953.)

E. W. Graham's method of drag reduction is used to study the effect of incidence on drag due to lift for wing-body combinations at supersonic speeds.—(1.10.2.2 × 1.2.3).

Determination of mean camber surfaces for wings having uniform chordwise loading and arbitrary spanwise loading in subsonic flow. S. Katzoff, M. Frances Faison and H. C. DuBose. N.A.C.A. Technical Note 2908 (*May* 1953).

Methods involving integrations around the wing boundary are presented for computing mean camber surfaces to support uniform chordwise loading, with either uniform or non-uniform spanwise loading, in subsonic flow. For polygonal wings with uniform area loading, analytical expressions are developed for both the slopes and the ordinates of the mean camber surfaces. Mean camber surfaces of several calculated wings are shown.—(1.10.1.2).

HELICOPTER AERODYNAMICS

The low speed performance of a helicopter. A. L. Oliver. A.R.C. Current Paper No. 122. (*May* 1952, *published* 1953.)

The analysis and estimation of helicopter performance is dependent upon the accurate assessment of rotor induced velocity. An empirical curve relating the flow through the rotor to the flight speed is used for vertical flight and the momentum theory is sufficiently accurate for a tip speed ratio greater than about 0·1, but no simple method has been generally available for the intermediary speed range. Sets of empirical curves covering this speed range and based on the analysis of low speed flight performance are given.—(1.11).

Effects of compressibility on the performance of two full-scale helicopter rotors. P. J. Carpenter. N.A.C.A. Report 1078.

An investigation has been conducted on the Langley helicopter test tower to determine experimentally the effects of compressibility on the performance and blade pitching moments of two full-scale helicopter rotors with twisted and untwisted blades over a tip-speed range from 350 to 770 ft./sec. (Formerly Technical Note 2277.)—(1.4).

An investigation of the experimental aerodynamic loading on a model helicopter rotor blade. J. R. Meyer, Jr., and G. Galabella, Jr. N.A.C.A. Technical Note 2953 (*May* 1953).

Pressure distributions were measured on a model helicopter rotor blade under hovering and simulated forward-flight conditions. Pressures were recorded at advance ratios of 0·10, 0·22, 0·30, 0·40 and 0·50 for a zero-offset flapping-hinge rotor and at 0·10, 0·22, 0·30, 0·45, 0·60, 0·80 and 1·0 for a lifting rotor with a flapping-hinge offset of 13 per cent.—(1.11).

TESTING AND INSTRUMENTS

The aerodynamic design and calibration of an asymmetric variable Mach number nozzle with a sliding block for the Mach number range 1·27 to 2·75. P. B. Burbank and R. W. Byrne. N.A.C.A. Technical Note 2921 (*April* 1953).

A method of designing an asymmetric, fixed geometry, variable Mach number nozzle has been developed by using the method of characteristics. A small nozzle conforming to the analytically determined ordinates was constructed and calibrated over a range of Mach numbers extending from 1·27 to 2·75. The results show the variation in Mach numbers to be ±0·02 or less and in the flow direction to be ±0·2° within the test section. The range of Mach numbers from 1·27 to 2·75 was obtained by translating the lower block in a straight line parallel to the test-section centre line for a distance of 2·17 test-section heights.—(1.12).

A new shadowgraph technique for the observation of conical flow phenomena in supersonic flow and preliminary results obtained for a triangular wing. E. S. Love and C. E. Grigsby. N.A.C.A. Technical Note 2950 (*May* 1953).

A new shadowgraph technique is presented for the observation of conical flow phenomena in supersonic flow in a plane normal or nearly normal to the axis of propagation. Preliminary results obtained for a triangular wing are also presented.—(1.12 × 1.10.2).

AEROELASTICITY

Single-degree-of-freedom-flutter calculations for a wing in subsonic potential flow and comparison with an experiment. H. L. Runyan. N.A.C.A. Report 1089 (1952).

The effect of Mach number and structural damping on single-degree-of-freedom pitching of a wing is presented. Some experimental results are compared with theory and good agreement is found for certain ranges of an inertia parameter. (Formerly Technical Note 2396.)—(2 × 1.8.2).

Flight investigation of the effect of transient wing response on wing strain of a four-engine bomber airplane in rough air. H. N. Murrow and C. B. Payne. N.A.C.A. Technical Note 2951 (*June* 1953).

The results of a flight investigation on a four-engine bomber aeroplane to determine the effect of wing flexibility on wing strains show that the average amplification at the wing root station for bending strains per *g* in gusts was approximately 31 per cent. higher than the bending strains per *g* in slow pull-ups. The amplification factor decreased in the outboard direction, except for the most outboard station where it increased slightly.—(2 × 31.1.2).

AIRCRAFT OPERATION

SAFETY

See also METEOROLOGY

Note on the flight testing and assessment of icing protection systems. D. Fraser. N.A.E. Canada, Laboratory Report LR-50 (*March* 1953).

Flight tests of an icing protection system consist of functioning tests, tests to determine the internal efficiencies, and tests of the performance in icing. The performance in icing can be determined only if the appropriate measurements are made, and if the flow of protection or the icing severity can be adjusted. The results can be reduced to standard conditions, using theoretical corrections, and can be plotted on a diagram to show whether performance meets design requirements, or to assess the degree of protection afforded by the ultimate, or any lower, performance.—(5.4).

Determination of means to safeguard aircraft from power plant fires in flight. Part V. The Lockheed Constitution (Navy XR60-1). L. E. Tarbell. C.A.A. Technical Development Report No. 198 (*April* 1953).

A full-scale operating XR60-1 Navy Constitution combination of a power plant, nacelle, and wing was subjected to fire tests under simulated flight conditions. Five separate investigations were conducted concerning: (1) fire detection, (2) fire extinguishment, (3) crew procedure, (4) materials and design, and (5) ignition sources.—(5.4).

Aircraft fire extinguishment. Part II. The effect of air flow on extinguishing requirements of a jet power-plant fire zone. C. A. Hughes. C.A.A. Technical Development Report No. 205 (*June* 1953).

Test fires were conducted in the fire zone of a jet power plant in order to determine the minimum amounts of fire-extinguishing agents required for extinguishment under different conditions of air flow through the zone. The tests were conducted with four extinguishing agents which were discharged through conventional and experimental systems; first, with the zone in its original condition, and later, with a lining in the lower portion of the fire zone to provide a smoother flow of air.—(5.4 × 27.1).

Impingement of water droplets on N.A.C.A. 65$_1$*-208 and 65*$_1$*-212 airfoils at 4° angle of attack. R. J. Brun, Helen M. Gallagher and Dorothea E. Vogt. N.A.C.A. Technical Note* 2952 (*May* 1953).

The trajectories of droplets in the air flowing past an N.A.C.A. 65$_1$-208 aerofoil and an N.A.C.A. 65$_1$-212 aerofoil, both at an angle of attack of 4°, were computed with a mechanical analogue. The amount of water in droplet form impinging on the aerofoils, the area of droplet impingement, and the rate of droplet impingement per unit area on the aerofoil surface affected were calculated.—(5.4 × 24).

ELECTRONICS

Development of a DME antenna transfer switch. J. R. Hoffman, R. E. Carlson and W. E. Haworth. C.A.A. Technical Development Report No. 201 (*May* 1953).

This report describes an antenna transfer switch developed at the Technical Development and Evaluation Centre of the Civil Aeronautics Administration for use with airborne distance measuring equipment (DME). This switch operates with two airborne antennas and one interrogator to minimise the loss of signal caused by interruption of the line of sight by a portion of the aircraft structure during turns in areas of low field strength. The locations of the aircraft antennas are chosen so that the line of sight to the ground equipment (transponder) can be maintained with at least one antenna while the aircraft is in any normal operating attitude. The switch was designed to operate with a modified Federal Telecommunication Laboratories DIA interrogator, but can be made to operate with any type of DME interrogator. —(11).

Optimum controllers for linear closed-loop systems. A. S. Boksenbom, D. Novik and H. Heppler. N.A.C.A. Technical Note 2939. (*April* 1953.)

An analysis is made for optimum controllers of general, linear, time-invariant multiloop systems based on minimising the mean squares or integral squares of errors and constrained variables for either stationary statistical or transient inputs. A general method of using open-loop optimum filter theory is derived that assures structural stability of the closed-multiloop system during the optimisation process. General solutions were obtained for certain special cases and applied to examples of turbojet engine control.—(11 × 27.1).

HYDRODYNAMICS

PLANING SURFACES

Hydrodynamic impact of a system with a single elastic mode. I—Theory and generalized solution with an application to an elastic airframe. W. L. Mayo. N.A.C.A. Report 1074 (1952).

Solutions of impact of a rigid prismatic float connected by a massless spring to a rigid upper mass are presented. The solutions are based on hydrodynamic theory which has been experimentally confirmed for a rigid structure. Equations are given for defining the spring constant and the ratio of the sprung mass to the lower mass so that the two-mass system provides representation of the fundamental mode of an aeroplane wing. (Formerly Technical Note 1398.)—(17.2 × 33.1.2).

Hydrodynamic impact of a system with a single elastic mode. II—Comparison of experimental force and response with theory. R. W. Miller and K. F. Merten. N.A.C.A. Report 1075 (1952).

Hydrodynamic impact tests were made on an elastic model approximating a two-mass-spring system which had a ratio

of sprung mass to hull mass of 0·6 and a natural frequency of 3·0 cycles per second. Tests were made at two combinations of trim and flight-path angles and for a range of flight-path velocity. Comparison of the experimental results with results obtained from the theory of N.A.C.A. Report 1074 showed good agreement. (Formerly Technical Note 2343.)—(17.2 × 33.1.2).

MATERIALS

Properties of Metallic Materials

*The fabrication and room temperature brittleness of chromium. F. Henderson and H. L. Wain. A.R.L. Australia, Report SM.*202 (*March* 1953).

Arc-melted chromium can be worked in the temperature range 600-900°C. by forging, swaging and rolling. It was found that chromium is inherently ductile at room temperature, and that the usual brittleness appears to be associated with specific impurities in the metal.—(21.2.2).

Combined-stress fatigue strength of 76S-T61 aluminum alloy with superimposed mean stresses and corrections for yielding. W. N. Findley. N.A.C.A. Technical Note 2924 (*May* 1953).

Fatigue data for 76S-T61 aluminium alloy are presented for several combinations of bending and torsion with both alternating and mean stresses. Special fatigue equipment for these tests is described. Correction for yielding was applied to the mean stresses in the bending and in the torsion fatigue tests. The literature on the effect of combined stress and of mean stress in fatigue is reviewed and the results of the present series are compared with those of various theories.—(21.2.2).

The creep of single crystals of aluminum. R. D. Johnson, F. R. Shober and A. D. Schwope. N.A.C.A. Technical Note 2945 (*May* 1953).

The creep of single crystals of high-purity aluminium was investigated in the range of temperatures from room temperature to 400°F. and at resolved-shear-stress levels of 200, 300 and 400 lb./in.^{2}. The tests were designed in an attempt to produce data regarding the relation between the rate of strain and the mechanism of deformation. The creep data are analysed in terms of shear strain rate and the results are discussed with regard to existing creep theories. Stress-strain curves were determined for the crystals in tensile and constant-load-rate tests in the same temperature range to supplement the study of plastic deformation by creep with information regarding the part played by crystal orientation, differences in strain markings, and other variables in plastic deformation.—(21.2.2).

Properties of Non-metallic Materials

The galvanic corrosion theory for adherence of porcelain-enamel ground coats to steel. D. G. Moore, J. W. Pitts, J. C. Richmond and W. N. Harrison. N.A.C.A. Technical Note 2935 (*June* 1953).

The galvanic corrosion theory of adherence between ground-coat enamels and steel was investigated. The theory, which is based on mechanical anchoring of the enamel into pits formed by galvanic attack of the enamel on the steel, was first examined from the standpoint of data on adherence obtained in earlier studies. Also, several experiments were performed which demonstrated that galvanic corrosion could occur during the short firing times encountered in enamel processing. However, inconsistencies were observed in the data which indicated that the mechanism of galvanic attack followed by mechanical anchoring was not the only important factor affecting bond strength.—(21.3.1).

METEOROLOGY

See also Aircraft Operation

*Meteorological design requirements for icing protection systems. D. Fraser. N.A.E. Canada, Laboratory Report LR-*49 (*March* 1953).

The meteorological parameters which affect various icing protection systems are considered, and these parameters are used to define basic severity and distribution requirements. The characteristics of various protective systems are then examined, and it is shown how the requirements should be applied, taking these characteristics into consideration. A specific application to an alcohol spray system is used to demonstrate how conformity to the requirements is judged, and to show how concessions from the requirements can be quantitatively evaluated.—(5.4 × 24).

POWER PLANTS

See also Electronics and Aircraft Operation

The attenuation characteristics of four specially designed mufflers tested on a practical engine setup. G. M. Stokes and D. D. Davis, Jr. N.A.C.A. Technical Note 2943 (*May* 1953).

Attenuation characteristics of four different resonator mufflers were determined in both cold tests and engine field tests and compared with the theoretical calculations. These mufflers were specifically designed for a helicopter. Engine-exhaust sound pressures, temperatures, and noise levels from the helicopter were measured.—(27.2.2).

PROPELLERS

Wake survey and strain-gauge measurements on an inclined propeller in the R.A.E. 24-ft. tunnel. Part I. Wake survey. J. G. Russell. A.R.C. Current Paper No. 117 (*February* 1952, *published* 1953).

This report describes tests made in the R.A.E. 24 ft. wind tunnel with a 16 ft. diameter, 4 bladed propeller. Wake survey and blade strain gauge measurements were made at tunnel speeds of 100 and 170 f.p.s. with the propeller axis inclined at angles of 0, 5, 10 and 15° to the air flow. The blade angles and propeller rotational speeds were also varied within the limits imposed by the 1,500 h.p. electric motor. The lift grading curves at the points of maximum and minimum loading, derived from total head measurements made in the slipstream by means of a pitot comb, have been compared with estimated values, and estimated power absorption figures with measured values.—(29.7 × 29.1).

RESEARCH

See also Aeroelasticity

General—Equipment

A small Pirani gage for measurements of nonsteady low pressures. M. J. Pilny. N.A.C.A. Technical Note 2946 (*June* 1953).

A small Pirani gauge made of surgical grain-of-wheat lamps and its operating equipment for the measurement of 12 channels of low pressures in the range 0·1 to 10 mm Hg abs are described, together with techniques of calibration and use. Theoretical and measured characteristics of small Pirani gauges are presented to show their suitability for the precise measurement of non-steady low pressures.—(31.1).

A variable-frequency light synchronized with a high-speed motion-picture camera to provide very short exposure times. W. F. Lindsey and J. Burlock. N.A.C.A. Technical Note 2949 (*May* 1953).

A new high-speed photographic technique has been developed which employs a variable-frequency light synchronised with a commercially available 16-millimeter high-speed motion-picture camera without appreciable alterations to the camera. The technique is described and results obtained by this technique of photographing the flow past models in a wind tunnel employing the Schlieren method of flow visualisation are presented. The photographs show that the new technique, through the use of extremely short exposure times (about 4 microseconds), provides more sharply defined pictures throughout the flow field than were obtained by conventional techniques.—(31.1).

STRUCTURES

See also Hydrodynamics

Theory and Analysis

Strength analysis of stiffened thick beam webs with ratios of web depth to web thickness of approximately 60. *L. R. Levin. N.A.C.A. Technical Note* 2930 (*May* 1953).

The results of an experimental investigation of the strength of plane diagonal-tension webs with ratios of web depth to web thickness of about 60 are presented. An analysis of the beams indicated that the methods of strength analysis presented in N.A.C.A. T.N. 2661 are applicable to beams with the flanges symmetrically arranged about the web if the portal-frame effect is taken into account.—(33.2.3).

APPOINTMENTS

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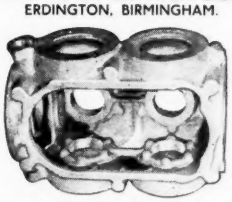

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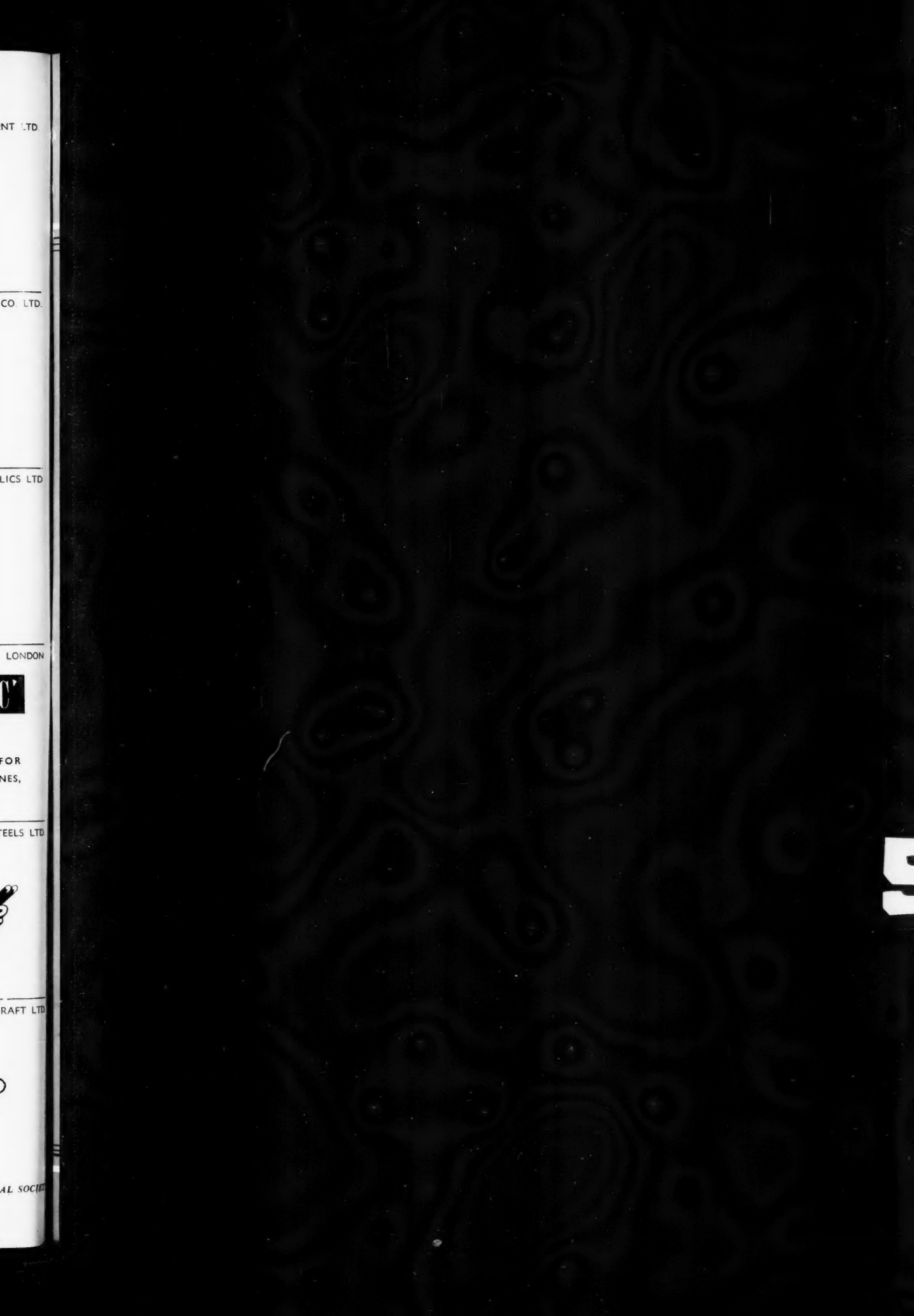
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